Quantum Context Free P Systems

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Abstract. Quantum computing and Membrane computing are two branches of Natural computing which have been studied extensively in recent years. In this paper we introduce the concept of quantum context free P systems which represents an intersecting combination of quantum grammars and P systems, thus providing a new novel approach towards the solution of complex computation problems.

Keywords: Quantum grammars· Context free grammars· Membrane computing· Amplitude· Quantum context free P systems.

1 Introduction

Quantum computing is a revolutionary concept in computation that leverages the principles of quantum mechanics to process and store information [1], [2], [3], [4]. Quantum automata and grammars are theoretical constructs that extend classical automata and grammars using the principles of quantum mechanics to perform complex computations much faster than their classical counterparts. Quantum grammars have been widely studied [5], [6], [7], [8] since the seminal paper of Moore and Crutchfield [9] where concepts from classical automata were extended to its quantum counterparts. In [9] the quantum version of a grammar was defined by adding an amplitude vector to each production.

Membrane computing is another field of natural computing that has been of interest recently [10], [11], [12] [13], [14]. It is a model which was inspired by the structure and functionalities of living cells. It provides a framework for studying and simulating complex biological processes and solving computationally hard problems. This model usually referred to as a P system was first introduced by Gheorghe Păun [12] and has been studied extensively [15, 16]. Rewriting P systems manipulate strings using rewriting rules. As a result these strings are capable of moving from one membrane to another.

Although the quantum p systems has been explored earlier, most studies based on quantum logic [17], [18], [19]. In contrast we define quantum context free p systems based on Moore's context free grammar. In our paper we use membrane computing to generate languages which are not context free using quantum context free production rules. The rest of this paper is organised as follows. In Section 2 the preliminary notions used to obtain our results are discussed. In Section 3 we combine the notions of quantum context free grammar and rewriting P system to define quantum context free P system. Finally we conclude in section 4.

2 Preliminaries

2.1 Quantum Grammars

Definition 1. *[9] A quantum grammar $G = (N, T, I, P)$ consists of two nonempty sets N and T, the set of nonterminals and the set of terminals respectively, an initial variable $I \in N$ and a finite set P of productions $\alpha \to \beta$ where $\alpha \in (N \cup T)^* N (N \cup T)^*$ and $\beta \in (N \cup T)^*$. Each production in P is associated with a set of complex amplitudes $c_k(\alpha \to \beta)$, $1 \leq k \leq n$ where n is the dimensionality of the grammar.*

Remark 1. This definition ensured the existence of a vector belonging to some vector space of dimension n corresponding to each production rule $p : (A \to \beta)$ in G. The notation $\mathbf{c}(A, \beta, G)$ or $c(p, G)$ was used in [20] to represent this vector.

We recall the following as in [20]:

Given strings w_1, w_2 and a production rule $p : A \to \gamma$, we associated the vector $\mathbf{c}(p)$ with the derivation $w_1 A w_2 \implies w_1 \gamma w_2$. Consider the derivation $\alpha = \alpha_0 \implies \alpha_1 \implies \alpha_2 \cdots \implies \alpha_m = \beta$ with amplitude vector $< c_{i_1}, c_{i_2}, \cdots c_{i_n} >$ corresponding to the i^{th} step, $\alpha_{i-1} \implies \alpha_i$ of the derivation. Then, the k^{th} amplitude c_k of the derivation $\alpha \overset{*}{\implies} \beta$ was defined as the product of the k^{th} amplitudes c_{i_k} for each production in the chain and $c_k(\alpha \overset{*}{\implies} \beta)$ as the sum of the $c_k^{'}s$ of all possible derivations of β from α. Then the k^{th} amplitude of a word $w \in T^*$ is $c_k(w) = c_k(I \overset{*}{\implies} w)$ and the probability associated with the generation of w is given by

$$f(w) = \sum_{k=1}^{n} |c_k(w)|^2$$

Then G generates the quantum language f.

Definition 2. *[9] A quantum grammar is context-free if the productions are of the form $\alpha \to \beta$, $\alpha \in N$.*

The following notations were also introduced :

1. The vector of amplitudes of the derivation given by

$$\alpha_0 \implies \alpha_1 \implies \cdots \implies \alpha_m$$

   is denoted by $\mathbf{c}(\alpha_0, \alpha_m)$.
2. The amplitude vector of the word w is denoted $\mathbf{c}(w)$.

3. $R(A) = \{\beta \in (N \cup T)^* | A \to \beta\}$ for $A \in N$.

Moreover we had extended the concept of amplitude to words over $(N \cup T)^*$.

As per the definition in [9], it is possible that the probability of derivation of certain words may exceed 1. To overcome this problem we introduced in [20] a restriction on the amplitude of each production defined in [9]. We then proved that under this restriction the function $f(w)$ defines a well-formed quantum language.

Theorem 1. *[20]*
Let $G = (N, T, I, P)$ be a quantum context free grammar such that

1. $\sum_{\beta \in R(A)} |\boldsymbol{c}(A, \beta)|^2 = 1$, *for each* $A \in N$.
2. *Given* $\beta, \gamma \in R(A)$, $\boldsymbol{c}(A \to \beta)$ *and* $\boldsymbol{c}(A \to \gamma)$ *are orthogonal.*

Then the function $f(w)$ defines a well-formed quantum language.

2.2 Membrane computing

A Rewriting P system [10] can be formally defined as

$$\Gamma = (N, T, \mu, w_1, \cdots, w_m, (R_1, \rho_1), \cdots, (R_m, \rho_m), i)$$

where

N is a non terminal alphabet
T is a terminal alphabet
μ is a membrane structure (of m membranes)
w_j, $1 \le j \le m$ is a word over $N \cup T$ (giving the initial content of membrane j)
R_j, $1 \le j \le m$ is a finite set of rules which can be applied to words in membrane j
ρ_j, $1 \le j \le m$ is a partial ordering on R_j
i is the output membrane.

3 Quantum Context Free P System

Definition 3. *A quantum context free P system of degree n, $n \ge 1$ is a construct* $\pi = (N, T, \mu, w_1, w_2, \cdots w_n, (R_1, \rho_1), (R_2, \rho_2), \cdots (R_n, \rho_n), i_0)$

N is a non terminal alphabet
T is a terminal alphabet
μ is a membrane structure
w_i, $1 \le i \le n$ are strings of objects over $N \cup T$ present in region i
$R_i, 1 \le i \le n$ is a finite set of context free rules of the form (r_{ij}, c_{ij}, tar) associated with the region i. Here $tar \in \{in_j, here, out\}$ and c_{ij} is the amplitude vector associated with r_{ij}

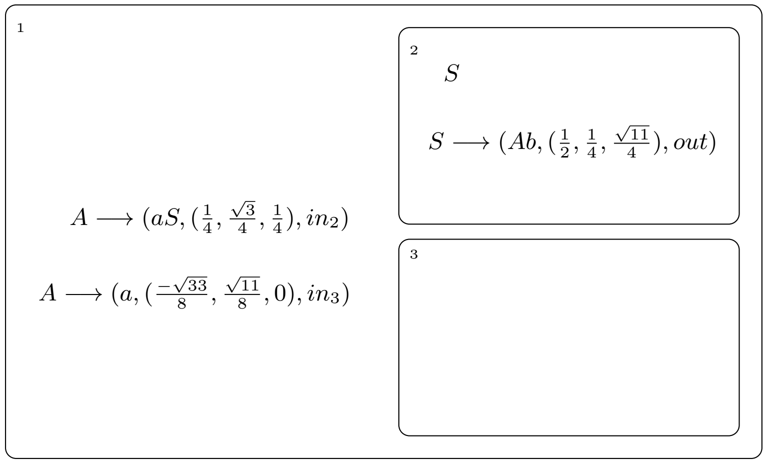

Fig. 1. A context free P System

ρ_i, $1 \le i \le n$ is a partial order relation over R_i
i_0 is an output membrane.

Example 1. Consider $\pi = (N, T, \mu, w_1, w_2, w_3, (R_1, \phi), (R_2, \phi), (R_3, \phi), 3)$

$N = \{S, A\}$
$T = \{a, b\}$
$\mu = [_1[_2]_2[_3]_3]_1$
$w_1 = \phi, R_1 = \{A \longrightarrow (aS, (\frac{1}{4}, \frac{\sqrt{3}}{4}, \frac{1}{4}), in_2),\ A \longrightarrow (a, (\frac{-\sqrt{33}}{8}, \frac{\sqrt{11}}{8}, 0), in_3)\}$
$w_2 = S,\ R_2 = \{S \longrightarrow (Ab, (\frac{1}{2}, \frac{1}{4}, \frac{\sqrt{11}}{4}), out)\},\ w_3 = \phi$

We apply the rule $S \longrightarrow Ab$ to get Ab with amplitude vector

$$\Big(\frac{1}{2}, \frac{1}{4}, \frac{\sqrt{11}}{4}\Big)$$

The result is moved to membrane 1. Next the rule $A \longrightarrow aS$ is applied to get aSb with amplitude vector

$$\Big(\frac{1}{8}, \frac{\sqrt{3}}{16}, \frac{\sqrt{11}}{16}\Big)$$

in membrane 2. After repeating $n-1$ steps the result is the string $a^{n-1}Sb^{n-1}$ in membrane 2 with amplitude

$$\Big(\Big(\frac{1}{4}\Big)^{n-1}\Big(\frac{1}{2}\Big)^{n-1}, \Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\Big(\frac{1}{4}\Big)^{n-1}, \Big(\frac{1}{4}\Big)^{n-1}\Big(\frac{\sqrt{11}}{4}\Big)^{n-1}\Big)$$

The rule $S \longrightarrow Ab$ is applied once again to obtain $a^{n-1}Ab^n$ with amplitude

$$\Big(\Big(\frac{1}{4}\Big)^{n-1}\Big(\frac{1}{2}\Big)^{n}, \Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\Big(\frac{1}{4}\Big)^{n}, \Big(\frac{1}{4}\Big)^{n-1}\Big(\frac{\sqrt{11}}{4}\Big)^{n}\Big)$$

in membrane 1. Finally the rule $A \longrightarrow a$ is applied to obtain $a^n b^n$ with amplitude

$$\Big(\frac{-\sqrt{33}}{8}\Big(\frac{1}{4}\Big)^{n-1}\Big(\frac{1}{2}\Big)^n, \frac{\sqrt{11}}{8}\Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\Big(\frac{1}{4}\Big)^n, 0\Big)$$

in the output membrane 3. The probability of accepting the word $a^n b^n$ is

$$\frac{11}{64}\frac{1}{4^{2n-2}}\Big[\frac{3}{2^{2n}} + \frac{(\sqrt{3})^{2n-2}}{4^{2n}}\Big] \leq 1$$

Theorem 2. *There exist languages, $L = \{w/f(w) > 0\}$ generated by quantum context-free P systems, which are not context-free.*

Proof. We give an example to illustrates this result.

Example 2. Consider $\pi = (N, T, \mu, w_1, w_2, w_3, (R_1, \phi), (R_2, \phi), (R_3, \phi), 3)$

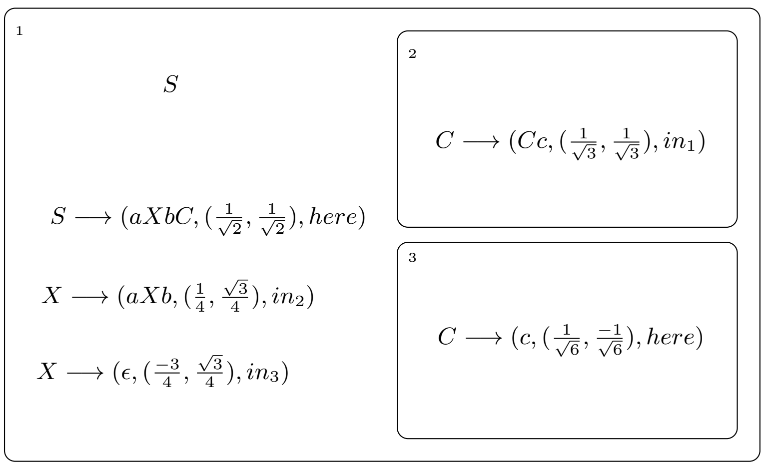

Fig. 2. A Context free P system generating a context sensitive language

$N = \{S, X, C\}$
$T = \{a, b, c\}$
$\mu = [_1[_2]_2[_3]_3]_1$
$w_1 = S, R_1 = \{S \longrightarrow (aXbC, (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}), here),\ X \longrightarrow (aXb, (\frac{1}{4}, \frac{\sqrt{3}}{4}), in_2),$
$X \longrightarrow (\epsilon, (\frac{-3}{4}, \frac{\sqrt{3}}{4}), in_3)\},\ \rho_1 = \phi$
$w_2 = \phi,\ R_2 = \{C \longrightarrow (Cc, (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), in_1)\},\ \rho_2 = \phi$
$w_3 = \phi,\ R_3 = \{C \longrightarrow (c, (\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}), here),\ \rho_3 = \phi$

We begin by applying the rule $S \longrightarrow aXbC$ with amplitude vector

$$\Big(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\Big)$$

to remain in membrane 1. We now apply $X \longrightarrow aXb$ which yields $aaXbbC$ in membrane 2 with amplitude vector

$$\Big(\frac{1}{4\sqrt{2}}, \frac{\sqrt{3}}{4\sqrt{2}}\Big)$$

.

In membrane 2 the rule $C \longrightarrow Cc$ is applied to obtain $aaXbbCc$ with amplitude vector

$$\Big(\frac{1}{4\sqrt{6}}, \frac{1}{4\sqrt{2}}\Big)$$

in membrane 1. After $n-1$ repetitions of the last two steps we obtain $a^n X b^n C c^{n-1}$ with amplitude vector

$$\Big(\Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{1}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}, \Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}\Big)$$

in membrane 1. Now in membrane 1 we apply the rule $X \longrightarrow \epsilon$ to get the string $a^n b^n C c^{n-1}$ with amplitude vector

$$\Big(\frac{-3}{4}\Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{1}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{4}\Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}\Big)$$

and moves to membrane 3. Finally the rule $C \longrightarrow c$ is applied in membrane 3 to get $a^n b^n c^n$ with amplitude

$$\Big(\frac{1}{\sqrt{6}}\frac{-3}{4}\Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{1}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{6}}\frac{\sqrt{3}}{4}\Big(\frac{1}{\sqrt{3}}\Big)^{n-1}\Big(\frac{\sqrt{3}}{4}\Big)^{n-1}\frac{1}{\sqrt{2}}\Big)$$

$$= \Big(-\frac{1}{2}\frac{1}{(\sqrt{3})^{(n-2)}}\frac{1}{4^n}, -\frac{1}{2}\frac{1}{4^n}\Big)$$

The probability of acceptance of the word $a^n b^n c^n$ is

$$\frac{1}{4^{2n+1}}\Big(1 + \frac{1}{3^{n-2}}\Big) \leq 1$$

It may be noted however that as n increases the probability of acceptance of the word $a^n b^n c^n$ decrease exponentially.

4 Conclusion

In this paper we have introduced the concept of a quantum context free P system and showed that it is more powerful than a quantum context free grammar. Possibilities for future work could include the study of other properties of these systems.

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