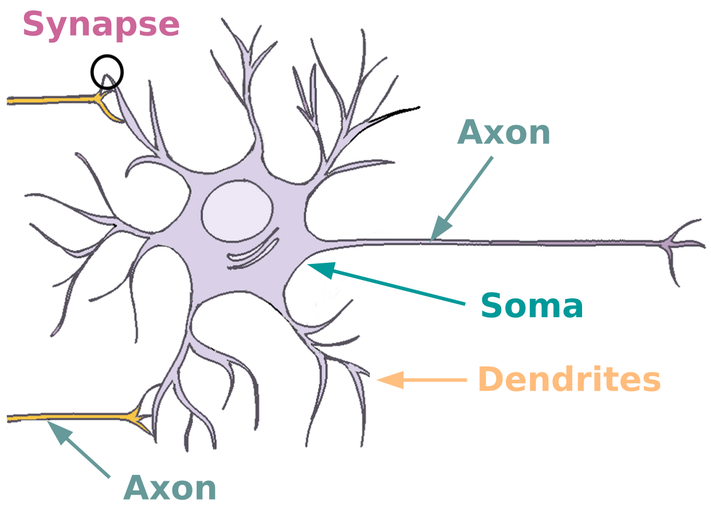
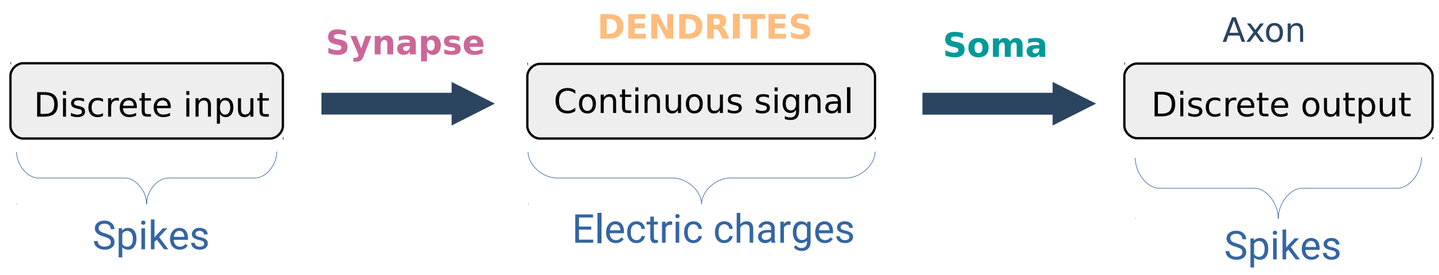
Dendritic Integration: an Algebraic Approach

O. Guinaudeau, G. Bernot, A. Muzy, F. Grammont

ophelie.guinaudeau@i3s.unice.fr - I3S - UMR7271 - UCA/CNRS

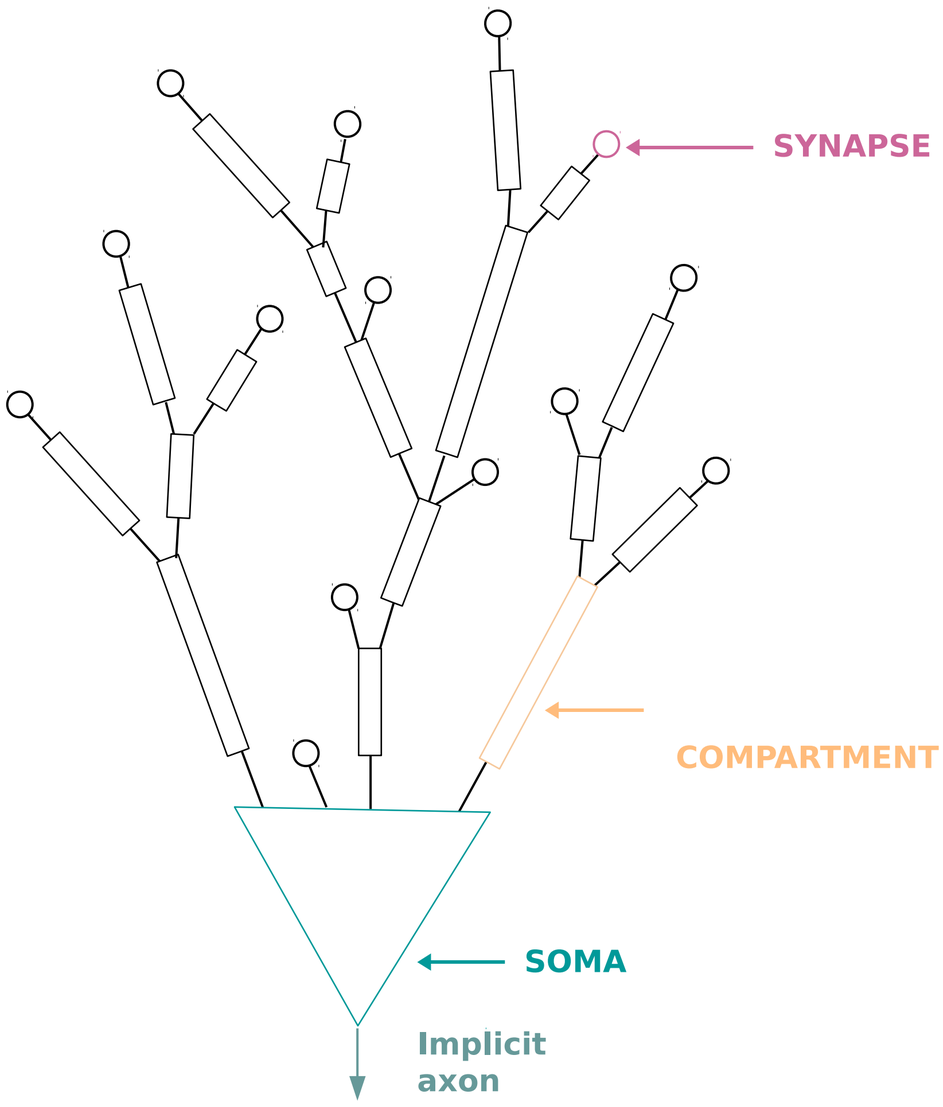


The biological neuron is a cell specialized in processing and transmission of information.

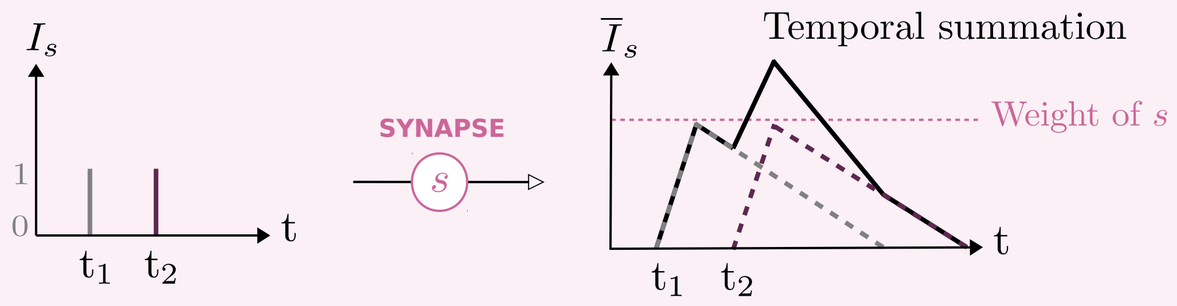


Objective: To extend the classical Leaky Integrate-and-Fire model (Lapicque, L., 1907) with dendritic integration, in a formal way, with nice algebraic properties.

NEURON = LABELLED ROOTED TREE



Input spikes are converted into local variations of the electric charge

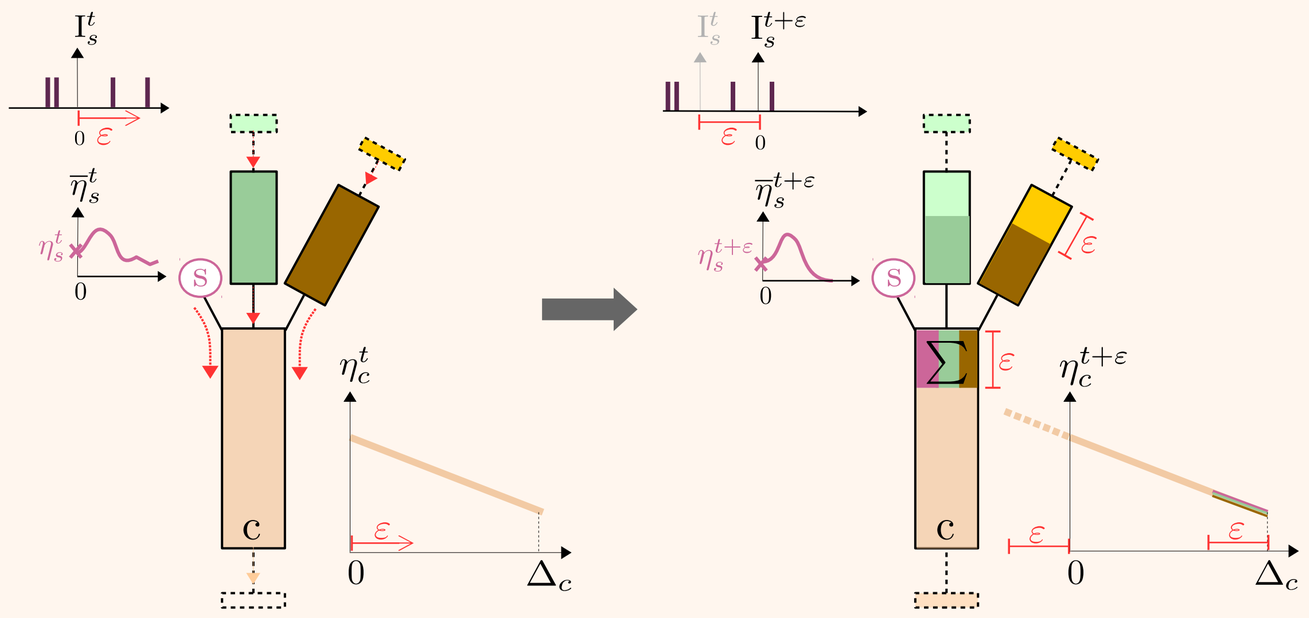


Charges propagate through the compartments

- Hypothesis of unidirectional & passive propagation
- Compartment c describes by only two parameters:

  Δ_c : delay for crossing c

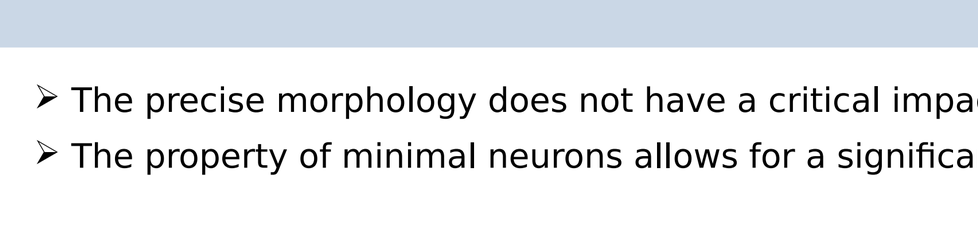
  α_c : attenuation at the end of c
- The state of c at a given time t (denoted η_c^t) is the electric charge at the end of c between times t and $t+\Delta_c$
- Dynamics computed by shifting the state by an interval of time ε



Charges accumulate at the soma + leak

- Inspired from Leaky Integrate-and-Fire model
- When reaching the nominal border:

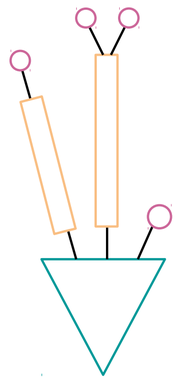
  Output spike + charge update + refractory period



ALGEBRAIC PROPERTIES

- **Neuron morphisms** preserve delays and attenuations from synapses to soma
- **Input signals** have a module structure on the ring $\mathbb{R}^{\text{synapses}}$
- **Dendritic states** have an abelian group structure

Based on these properties, **category theory** helps proving that for all morphisms $\mu : N \to N'$, the neurons N and N' share the same **input/output function**



Minimal objects of the category of neurons are normalized neurons with:

- Height of tree ≤ 1
- No attenuation in compartments, for all compartment $\alpha_c = 1$
- Normal threshold = 1

CONCLUSION

- The precise morphology does not have a critical impact on the neuron input/output function, only **delays and synaptic weights** matter
- The property of minimal neurons allows for a significant computational **reduction of complexity** when simulating neuronal networks

Bleu foncé: 2b445f

Bleu clair: cad7e6

Bleu très clair: e8f1f6

Rose: a0789a

Orange: ffbc7c