Good functors ... are those preserving philosophy !

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Abstract

The aim of this paper is to prevent the abstract data type researcher from an improper, naive use of category theory. We mainly emphasize some unpleasant properties of the *synthesis functor* when dealing with so-called *loose semantics* in a hierarchical approach. All our results and counter-examples are very simple, nevertheless they shed light on many common errors in the abstract specification field.
We also summarize some properties of the category of models "protecting predefined sorts."

Keywords : abstract data types, abstract specifications, category theory, completeness, consistency, initial model, structured specifications.

1 Introduction

In the following pages, we focus our attention on results which seem to be "trivially ensured" in the basic abstract data type framework. We sometimes give proofs... often counter-examples of such results. In order to get striking counter-examples, we provide very simple ones, if not trivial (mainly based on elementary algebraic properties of natural numbers). Nevertheless, many common errors, or misinterpretations found in the abstract data type litterature result from similar mechanisms. This emphasizes the fact that category theory should be carefully used in the abstract data type field, including for (very) low level concepts.
More provocatively: this paper mainly points out the fact that the synthesis functor $\mathcal{F}$ of abstract data types "does not preserve philosophy." However, since about teen years [ADJ76], it is well known that this functor is crucial for defining a hierarchical, modular approach of abstract specifications!

Some elementary reminders about abstract data types are given in the next section (Section 2). Section 3 discusses about the well known *forgetful* and *synthesis functors*, $\mathcal{U}$ and $\mathcal{F}$, associated with a hierarchical approach. Section 4 shows the difficulty of properly defining sufficient completeness and hierarchical consistency with loose semantics. In Section 5, we show what happens when combining enrichments. Lastly, Section 6 discusses about a loose semantics obtained by "protecting" predefined sorts.

The following discussions are mainly centered on pairs [positive fact / proof] (respectively: [negative fact / counter-example]).

2 Elementary reminders

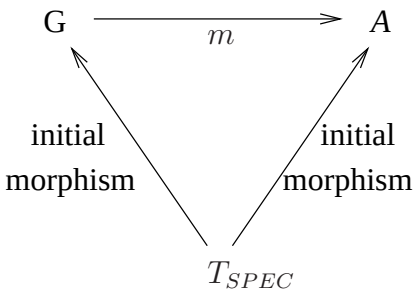
Let us begin with basic definitions and properties [ADJ76]:
Given a *signature* Σ (i.e. a finite set S of *sorts* and a finite set Σ of *operation-names* with arity in S), a Σ-algebra, A, is a heterogeneous set partitioned as $\{ A_s \}_{s \in S}$, and for each operation-name $op : s_1 \cdots s_{n-1} \rightarrow s_n$ of Σ there is an operation $op_A : A_{s_1} \times \cdots \times A_{s_{n-1}} \rightarrow A_{s_n}$. A Σ-*morphism* from A to B is a sort-preserving, operation-preserving application from A to B. This defines a category, denoted by $Alg(\Sigma)$; it has an initial object: the ground-term algebra T_Σ .
In the following, a *specification* $SPEC$ will be defined by a signature Σ and a finite set E of *positive conditional equations* of the form:

$$v_1 = w_1 \ \wedge \cdots \wedge \ v_{n-1} = w_{n-1} \Longrightarrow v_n = w_n$$

where v_i and w_i are Σ-terms with variables.
Given a specification $SPEC$, $Alg(SPEC)$ is the full sub-category of $Alg(\Sigma)$ whose objects are the Σ-algebras which validate each axiom of E. The category $Alg(SPEC)$ has an initial object, denoted by T_{SPEC} [BPW82].
Since T_{SPEC} exists, $Gen(SPEC)$ can be defined as the full sub-category of $Alg(SPEC)$ such that the initial morphism is an epimorphism (i.e. is *surjective*, in our framework). $Gen(SPEC)$ is the category of the *finitely generated algebras*. Our first "fact" will be devoted to the following remark:
It is well known that $Gen(SPEC)$ is a particularly interesting category for the abstract data type computer scientist; nevertheless, this is not exactly due to its large spectrum of morphisms, as reminded below.

fact 1 : (Morphisms from a finitely generated algebra)
Let Γ be an object of $Gen(SPEC)$ and A an object of $Alg(SPEC)$. The set $Hom_{Alg(SPEC)}(\Gamma, A)$ contains *at most one element*. Consequently, for all objects X and Y of $Gen(SPEC)$, $Hom_{Gen(SPEC)}(X, Y)$ contains at most one morphism.

Proof: By initiality properties, if there exists a morphism μ, then the following triangle commutes:



Thus, the unicity of μ results from the surjectivity of the initial morphism associated with Γ. □

One of the most important aspect of abstract data types is its structured, hierarchical, modular approach. This is obtained by means of *presentations*. A presentation $PRES$ over $SPEC$ is a new "part of specification" $PRES =< S', \Sigma', E' >$ such that the disjoint union $SPEC + PRES =< S \cup S' \ , \ \Sigma \cup \Sigma' \ , \ E \cup E' >$ is a specification. Sorts and operations of $SPEC$ are often called the *predefined* sorts and operations. Relations between the categories $Alg(SPEC)$ and $Alg(SPEC + PRES)$ are handled by the well known *forgetful functor* and *synthesis functor*:

$$(\mathcal{U} \ : \ Alg(SPEC + PRES) \rightarrow Alg(SPEC)) \text{ and } (\mathcal{F} \ : \ Alg(SPEC) \rightarrow Alg(SPEC + PRES)).$$

The functor $\mathcal{F}$ is a left adjoint for the functor $\mathcal{U}$. Consequently, for each $SPEC$-algebra A, there is a particular morphism from A to $\mathcal{U}(\mathcal{F}(A))$: the morphism deduced from the adjunction unit (or *adjunction morphism*). This morphism is absolutely crucial for the hierarchical approach: it allows to evaluate the modifications performed on A under the action of $PRES$.

Example 2 : If A is equal to $\mathbb{N}$ over the signature $\{0, succ\_\}$ (without axioms) and if $PRES$ adds $pred\_$ with the axioms $[pred(succ(n)) = succ(pred(n)) = n]$, then $\mathcal{U}(\mathcal{F}(\mathbb{N}))$ is isomorphic to $\mathbb{Z}$. The unit of adjunction leads to the natural inclusion; and this morphism permits to show that $\mathbb{N}$ has been modified by adding negative values.
If the axioms were $[pred(succ(n)) = n$ and $pred(0) = 0]$, then the unit of adjunction leads to the identity over $\mathbb{N}$ showing that this second specification of $pred$ does not change $\mathbb{N}$.

3 Forgetful and synthesis functors

We first present a rather obvious reminder about the *forgetful functor*. Let B be a $SPEC+PRES$-algebra. The forgetful functor removes all subsets B_s where $s \in S'$, and all operations of Σ' are forgotten (including those with arity in S only), but *it does not remove any value of predefined sort*: $\mathcal{U}(B_s) = B_s$ for each $s \in S$. For instance, in Example 2, $\mathcal{U}(\mathbb{Z}) = \mathbb{Z} \neq \mathbb{N}$.

Let us remind the classical definition of the *synthesis functor* (although classical, this definition is the starting point of some misinterpretations!): Let A be a $SPEC$-algebra and let $T_{\Sigma+\Sigma'}(A)$ be the algebra of $\Sigma + \Sigma'$-terms with variables in A; we denote by $eval : \mathcal{U}(T_{\Sigma+\Sigma'}(A)) \rightarrow A$ the canonical evaluation morphism. $\mathcal{F}(A)$ is the quotient of $T_{\Sigma+\Sigma'}(A)$ by the smallest congruence containing both the fibers of $eval$ and the close instanciations of $E + E'$.
Because $E + E'$ is required in the definition of $\mathcal{F}$ (instead of E' alone), $\mathcal{F}(A)$ does not only depend on A and $PRES$; it also depends on $SPEC$.

fact 3 : Given a presentation $PRES$, the action of the synthesis functor $\mathcal{F}$ over a given, fixed algebra A is highly dependent of the predefined specification.
As outlined in the following example, this fact considerably restricts the possibility of writing "implementation independent" specifications (see for instance [EKMP80], [SW82], or [BBC86a] about abstract implementations).

Example 4 : Let $SPEC$ be a classical specification of NAT with operations 0, $succ\_$ and $\_ + \_$:

$x + 0 = x$
$x + succ(y) = succ(x + y)$

Let $SPEC'$ be the specification obtained by adding the following axiom to $SPEC$:

$x + y = x + z \Longrightarrow y = z$

The specifications $SPEC$ and $SPEC'$ have clearly the same initial object: $\mathbb{N}$.
Let $PRES$ be the presentation adding no sort, adding the operation $\_ \times \_$, and adding the axioms:

$x \times succ(0) = x >$ (1 is neutral)
$x \times succ(y) = x + (x \times y) >$ (recursive definition)

When $PRES$ is shown as a presentation over $SPEC$, $\mathcal{F}(\mathbb{N})$ is a model where all terms containing a multiplication by 0 cannot be evaluated. When $PRES$ is shown as a presentation over $SPEC'$, $\mathcal{F}(\mathbb{N})$ is isomorphic to $\mathbb{N}$, because:

$$x + 0 = x = x \times succ(0) = x + (x \times 0)$$

and the simplification axiom of $SPEC'$ leads to $0 = x \times 0$.
Notice that, in spite of the fact that $SPEC$ and $SPEC'$ have the same initial semantics, the presentation $PRES$ is not completely specified over the first specification, but is completely specified over the second one.

4 Consistency and completeness

The subject of this section is an examination of some *a priori* possible definitions of the notions of *sufficient completeness* and *hierarchical consistency* with loose semantics. We start with the most loose semantics: the entire category $Alg(SPEC)$. We will show that the simplest definitions are unacceptable for abstract specification purposes.

All the counter-examples provided in this section are based on the following specification+presentation example. Hopefully, we believe that this counter-example cannot be suspected to be too much unusual, complicated or *ad hoc*.

Example 5 : Let $SPEC$ be a specification of natural numbers (for instance the specification given in Example 4) together with a sort $BOOL$ and boolean operations $True$ and $False$. We consider the presentation $PRES$ enriching $SPEC$ by an equality predicate $eq?$:

$$eq?(0, 0) = True$$
$$eq?(0, succ(n)) = False$$
$$eq?(succ(m), 0) = False$$
$$eq?(succ(m), succ(n)) = eq?(m, n)$$

Looking at this presentation $PRES$, we can affirm that a "good notion" of sufficient completeness (resp. hierarchical consistency) should be satisfied by $PRES$. This example is simply written by taking into account each possible value for the arguments of $eq?$, with respect to the constructors of $SPEC$, moreover there are no axioms between constructors (fair presentation [Bid82]).

We may of course imagine more sophisticated presentation examples, in particular examples which add new sorts to $SPEC$. But our goal is simply to prevent the abstract data type researcher from using a naive, rather unrealistic definition of sufficient completeness or hierarchical consistency.

4.1 Sufficient completeness

In the initial approach, sufficient completeness is defined as follows [Gau78].

"*The adjunction morphism associated with the initial algebra is surjective:*"

$$T\_SPEC \rightarrow \mathcal{U}(\mathcal{F}(T\_SPEC))$$

This condition exactly means that $PRES$ does not add new values to T_SPEC . Remind that $\mathcal{F}(T\_SPEC) = T_{SPEC+PRES}$, due to adjunction properties.

fact 6 : The following definition of sufficient completeness is not suitable in the general case: "$PRES$ is sufficiently complete if and only if for all algebras in $Alg(SPEC)$ the adjunction morphism is surjective".

Using Example 5, we convince ourselves of this fact by considering the $SPEC$-algebra obtained by two copies of $\mathbb{N}$. This algebra, ($\mathbb{N} \times \{0,1\}$ and $\{True, False\}$) , is not finitely generated, but is an object of $Alg(SPEC)$ by sending the operation-name 0 over the element $(0,0)$, and $succ((n,a)) = (succ(n), a)$. Terms of the form $eq?((n,0),(m,1))$ cannot be evaluated using the $PRES$ axioms of Example 5. Consequently, they add new boolean values, and the adjunction morphism is not surjective.

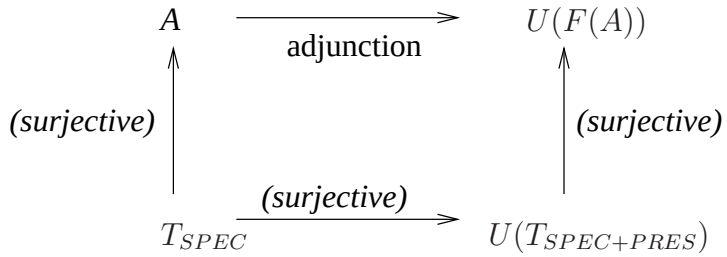
fact 7 : The following two definitions of sufficient completeness are logically equivalent:

1. the adjunction morphism associated with the initial algebra T_SPEC is surjective
2. for all algebras in $Gen(SPEC)$ the adjunction morphism is surjective.

Proof: $[2 \Longrightarrow 1]$ is trivial because the initial algebra is finitely generated.
$[1 \Longrightarrow 2]$: let A be a finitely generated $SPEC$-algebra. By construction of $\mathcal{F}$, $\mathcal{F}(A)$ is finitely generated over the signature of $SPEC + PRES$. Consequently, the image of the initial morphism via the forgetful functor is surjective:

$$\mathcal{U}(init_A) : \mathcal{U}(\mathcal{F}(T\_SPEC)) = \mathcal{U}(T_{SPEC+PRES}) \rightarrow \mathcal{U}(\mathcal{F}(A))$$

Our conclusion results from the commutativity of the following diagram:



□

Restricting ourselves to finitely generated algebras has several disadvantages. For instance, parameterized presentations require a non finitely generated semantics [ADJ80].

4.2 Hierarchical consistency

In the initial approach, hierarchical consistency is defined as follows:

"*the adjunction morphism associated with the initial algebra is a monomorphism*"

(i.e. is *injective* in our framework).

fact 8 : The following definition of hierarchical consistency is not suitable in the general case: "$PRES$ is hierarchically consistent if and only if for all algebras in $Alg(SPEC)$ the adjunction morphism is injective".

Let us return to Example 5. If we consider the $SPEC$-algebra $\mathbb{Z}$ (which is a non finitely generated algebra), we get the following inconsistency:

$$True = eq?(0,0) = eq?(0, succ(-1)) = False$$

Restricting hierarchical consistency checks to finitely generated algebras does not yield better results:

fact 9 : The following definition of hierarchical consistency is not suitable in the general case: "$PRES$ is hierarchically consistent if and only if for all algebras in $Gen(SPEC)$ the adjunction morphism is injective".

Using Example 5 again, we consider a finitely generated algebra of the form $\frac{\mathbb{Z}}{n\mathbb{Z}}$, and we get the following inconsistency:

$$True = eq?(0,0) = eq?(0,n) = eq?(0, succ(n-1)) = False$$

These facts prove that "defining sufficient completeness on $Alg(SPEC)$" , "defining hierarchical consistency on $Alg(SPEC)$" or "defining hierarchical consistency on $Gen(SPEC)$" are too strong requirements. Extension from the purely initial semantics to a loose semantics must be done more carefully.

5 Combining presentations

In the remainder of this paper, we simply follow the definitions of sufficient completeness and hierarchical consistency given at the beginning of sections 4.1 and 4.2 (i.e. the initial approach). Given a specification $SPEC$, we consider two presentations $PRES_1$ and $PRES_2$ *with disjoint signatures.*
Let $PRES$ be the union of $PRES_1$ and $PRES_2$, we care about the sufficient completeness and hierarchical consistency of $PRES$. In spite of the strong hypothesis described here, we have sometimes to be careful, as detailed in the following two subsections.

5.1 Sufficient completeness

fact 10 : If $PRES_1$ and $PRES_2$ are both sufficiently complete over $SPEC$, then $PRES = PRES_1 + PRES_2$ remain sufficiently complete. Moreover, under the same hypothesis, $PRES_2$ is sufficiently complete over $SPEC + PRES_1$.

Proof: *(using elementary tools)*
$T_{SPEC+PRES_1+PRES_2}$ is the quotient of $T_{\Sigma+\Sigma_1+\Sigma_2}$ by the smallest congruence containing the close instanciations of the $SPEC + PRES_1 + PRES_2$ axioms [BPW82]. Consequently, it suffices to prove that each $\Sigma + \Sigma_1 + \Sigma_2$-ground-term of sort in S (resp. in $S + S_1$) belongs to the equivalence class of a Σ-term (resp. $\Sigma + \Sigma_1$-term). This can be trivially proved via structural induction. □

Obviously, the converse is false: the sufficient completeness of $PRES$ does not imply the sufficient completeness of $PRES_1$ or $PRES_2$.

5.2 Hierarchical consistency

fact 11 : The hierarchical consistency of $PRES_1$ and $PRES_2$ over $SPEC$ *does not imply* the hierarchical consistency of $PRES = PRES_1 + PRES_2$ over $SPEC$.

Example 12 : Let *SPEC* be a specification of natural numbers. Let $PRES_1$ be the presentation simply containing the following axiom:

$$succ(n) = 0 \Longrightarrow n = 0$$

$PRES_1$ is clearly consistent (in fact, the premise cannot be satisfied in the initial object, thus this axiom is never applied). Let $PRES_2$ be the presentation adding the operation $pred\_$ with $[pred(succ(n)) = succ(pred(n)) = n]$. $PRES_2$ is clearly hierarchically consistent over natural numbers (even though it is not sufficiently complete). The union $PRES = PRES_1 + PRES_2$ is not hierarchically consistent because from $succ(pred(0)) = 0$ we get:

$$0 = pred(0) \text{ , which leads to } succ(0) = succ(pred(0)) = 0$$

Another example of the same fact is the following:

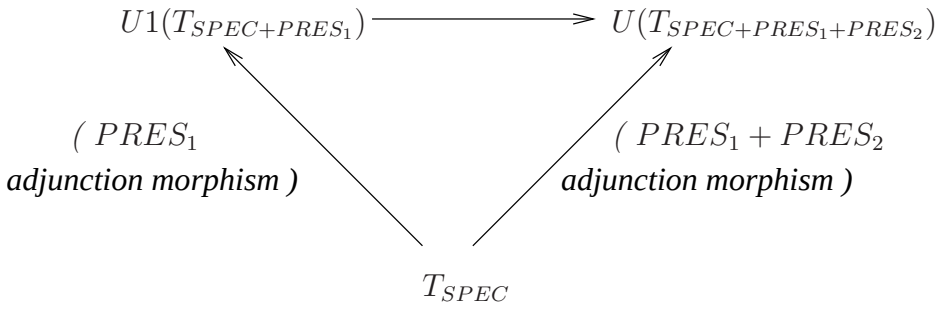
Example 13 : Let $PRES_1$ be the presentation described in Example 5 (adding equality predicate to natural numbers), and let $PRES_2$ be the same presentation as Example 12 before (adding *pred*). $PRES_1$ and $PRES_2$ are clearly hierarchically consistent over natural numbers, but the union $PRES = PRES_1 + PRES_2$ is not hierarchically consistent because:

$$True = eq?(0,0) = eq?(0, succ(pred(0))) = False$$

(a similar example was first presented in [EKP80], for abstract implementation purposes).

fact 14 : If $PRES = PRES_1 + PRES_2$ is hierarchically consistent over *SPEC* then $PRES_1$ and $PRES_2$ are hierarchically consistent over *SPEC*.

Proof: Assume that $PRES_1$ is not consistent: the morphism from T_{SPEC} to $\mathcal{U}(T_{SPEC+PRES_1})$ is not injective. Since the following diagram commutes, the adjunction morphism from T_{SPEC} to $T_{SPEC+PRES_1+PRES_2}$ is not injective:



(the horizontal arrow is the forgetful of the adjunction morphism for $PRES_2$ over $SPEC + PRES_1$).
It results that $PRES$ is not hierarchically consistent over *SPEC*. □

fact 15 : If $PRES_1$ and $PRES_2$ are both hierarchically consistent and sufficiently complete over *SPEC*, then $PRES = PRES_1 + PRES_2$ too. Moreover, under the same hypothesis, $PRES_2$ is hierarchically consistent and sufficiently complete over $SPEC + PRES_1$.

(This fact is well known; a demonstration with conditional axioms, including exception handling, can be found in [Ber86]).

6 Loose semantics with "Protect"

Clearly, abstract specifications do not necessarily directly lead to executable specifications. It is often convenient to specify some operations via "universal properties." For instance the subtraction can be specified via:

$$z - y = x \Longleftrightarrow x + y = z$$

Sometimes, such axioms may lead to uncompletely specified presentations, as in the following example.

Example 16 : Let $SPEC$ be an initial specification of integers with operations 0, $succ\_$, $pred\_$, $\_ + \_$, $\_ - \_$ and $\_ \times \_$. Let us specify a presentation $PRES$ adding the operation $\_div\_$ as follows:

$$0 \Longleftarrow (a - (b \times (adivb))) = True$$

$$(a - (b \times (adivb))) < b = True$$

These axioms characterize $(adivb)$ among all integers *finitely generated* with respect to *succ* and *pred*. However, in the initial model $T_{SPEC+PRES}$, the term $(adivb)$ is not reached by *succ* and *pred* . Its value is only a unreachable value such that the (unreachable) remainder $(a - (b \times (a\ div\ b)))$ returns the specified boolean values when compared with 0 and b . Consequently, this presentation is uncompletely specified according to the usual definition of sufficient completeness.

In such examples, the only interesting models are those which do not modify the predefined initial model ($\mathbb{Z}$). This leads to a (loose) semantics where models are those *protecting* predefined sorts [Kam80]. Indeed, when writing relatively large specifications, this semantics seems to be highly suitable (ASL [Wir82] [SW83], PLUSS [Gau84], OBJ [FGJM85], LARCH [GH83]...).

Let us define the associated category:

Definition 17 : (The "Protect" category)
Let $SPEC$ be a specification and let $PRES$ be a presentation over $SPEC$. The category of $PRES$-models protecting $SPEC$ is the full subcategory of $Alg(SPEC + PRES)$ whose objects are the $SPEC + PRES$-algebras A such that $\mathcal{U}(A)$ is isomorphic to the initial predefined algebra T_{SPEC} . We denote this category by $Prot(SPEC, PRES)$.

Notice that the object class of $Prot(SPEC, PRES)$ can be empty.

fact 18 : If $Prot(SPEC, PRES)$ is not an empty category, then $PRES$ is hierarchically consistent over $SPEC$.

(Here, consistency is defined with respect to the initial algebra T_{SPEC} only)

Proof: If $T_{SPEC+PRES}$ is inconsistent over T_{SPEC}, then a fortiori all $SPEC + PRES$-algebras are inconsistent over T_{SPEC} (because $T_{SPEC+PRES}$ is minimal). □

fact 19 : Even if $PRES$ is consistent over $SPEC$, $Prot(SPEC, PRES)$ may be empty.

Example 20 : Let $SPEC$ be the boolean specification with $True$ and $False$. Let $PRES$ be a specification of $SET(BOOL)$ with $\emptyset$, $insert$, $\in$ and $choose$:

$True \in \emptyset = False$
$False \in \emptyset = False$
$b \in insert(b', X) = (b{=}b') \; or \; b \in X$
$choose(X) \in X = True$

This specification is clearly hierarchically consistent (even though it is not sufficiently complete). However, the Protect category is empty, because the term $choose(\emptyset)$ can neither be equal to $True$ nor to $False$ (both choices induce $True = False$).
(Fortunately, this example can be easily specified without inconsistency using abstract data types with exception handling [Bid84] [GDLE84] [BBC86b] [Ber86], or with partial functions [BW82].)

fact 21 : Even if $Prot(SPEC, PRES)$ contains models, it has not necessarily an initial object.

Example 22 : Let $SPEC$ be the boolean specification with $True$ and $False$. Let $PRES$ be the presentation adding the constant operation *maybe*, without any axiom. $Prot(SPEC, PRES)$ contains two models, no one is initial.

fact 23 : If $PRES$ is sufficiently complete over $SPEC$, then either the category $Prot(SPEC, PRES)$ has an initial object, either it is empty.

Proof: If $PRES$ is consistent, then the initial model $T_{SPEC+PRES}$ belongs to $Prot(SPEC, PRES)$; it is then necessarily initial in $Prot(SPEC, PRES)$. If $PRES$ is not hierarchically consistent, then Fact 18 implies that $Prot(SPEC, PRES)$ has no object. □

fact 24 : There are presentations $PRES$ which are not sufficiently complete over $SPEC$, such that $Prot(SPEC, PRES)$ is not empty and has an initial object.

It suffices to refer to Example 16, where the axioms characterize *div* by a "universal property among integers." The division is incompletely specified according to classical initial definition of sufficient completeness, but $Prot(SPEC, PRES)$ only contains one model ($\mathbb{Z}$) which is necessarily initial.

7 Conclusion

We have investigated how a hierarchical approach of abstract data types, with the notions of *hierarchical consistency* and *sufficient completeness*, could be defined when dealing with so-called *loose semantics*. The results shown in sections 2 to 5 seem to be somewhat pessimistic:

- The synthesis functor is "implementation dependent" with respect to the predefined specification (Fact 3).
- Sufficient completeness cannot be checked on all models (Fact 6).
- Hierarchical consistency cannot be checked on all models (Fact 8).
- Hierarchical consistency cannot be checked on all finitely generated models, a smaller class of models must be investigated (Fact 9).
- Combining hierarchically consistent presentations does not result on a hierarchically consistent presentation (Fact 11).

However, we showed some positive results:

- Checking sufficient completeness on all finitely generated algebras is equivalent to check it on the initial algebra only (Fact 7).
- Combining sufficiently complete presentations results on sufficiently complete presentations (Fact 10); the same occurs for presentations that are both sufficiently complete and hierarchically consistent (Fact 15).

In the last section (Section 6), we defined the category of models *protecting predefined sorts*. We have investigated the relations between the classical notions of completeness/consistency and the elementary properties of this category:

- The category is empty if the presentation is not hierarchically consistent, but the converse is false (Facts 18 and 19).
- The category has not necessarily initial models (Fact 21).
  It has initial models if the presentation is sufficiently complete and hierarchically consistent, but the converse is false (Facts 23 and 24).

In conclusion: From facts 3, 6, 8, 9 and 11, we showed that the synthesis functor of classical abstract data types "does not always preserve philosophy" when dealing with loose semantics. Moreover, with a loose semantics based on protection of predefined sorts, the corresponding category has few systematic relations with sufficient completeness or hierarchical consistency (facts 18 to 24).

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