MULTIRESOLUTION 3D MESH COMPRESSION

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ABSTRACT

In this paper, we propose an efficient low complexity compression scheme for densely sampled irregular 3D meshes. This scheme is based on 3D multiresolution analysis (3D Discrete Wavelet Transform) and includes a model-based bit allocation process across the wavelet subbands. Coordinates of 3D wavelet coefficients are processed separately and statistically modeled by a generalized Gaussian distribution. This permits an efficient allocation even at a low bitrate and with a very low complexity. We introduce a predictive geometry coding of LF subbands and topology coding is made by using an original edge-based method. The main idea of our approach is the model-based bit allocation adapted to 3D wavelet coefficients and the use of EBCOT coder to efficiently encode the quantized coefficients. Experimental results show compression ratio improvement for similar reconstruction quality compared to the well-known PGC method [1].

1. INTRODUCTION

Triangular meshes are a powerfool tool for modeling the shape of complex 3D objects. Because of their simplicity (points and edges), they are easily manipulated and more and more present in 3D models visualisation setting. Triangular meshes often result from 3D acquisition techniques: they are finely detailled and highly sampled. Unfortunately they are very complex (irregular connectivity) and have tremendous size. Hence, they are awkward for computation, storage or transmission. Compression techniques are essential but visual quality must be preserved. The goal of compression algorithms is to strongly reduce the quantity of data to represent an object for a given global quality. In 2D image compression, the tools are well developed since decades and algorithms are now very efficient [2]. However, compression of 3D meshes are relatively new. Generally, they involve geometric and topologic data compression, and the limit of these kind of methods can be found in [3].

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Recently, Lounsbery [4] and Sweldens [1] introduced progressive compression schemes using 3D multiresolution analysis. These methods are based on the rate-distortion theory. Our framework is based on multiresolution analysis theory like in [4, 1] instead of a lossless non progressive compression like [5, 6].

This paper is organized as follows. Section 2 introduces the compression scheme and the models used by the bit allocation process. Section 3 deals with geometry coding while section 4 proposes a new algorithm for topology coding. Finally, we compare our algorithm with the PGC method and conclude in section 5.

2. 3D MULTIRESOLUTION SCHEME

2.1. Backgrounds

The first step of our compression scheme (see figure 1) is to obtain a semi-regular mesh of the original irregular mesh. The technique used is MAPS [7]. Hence, a Discrete Wavelet Transform (DWT) can be applied on the semi-regular mesh to obtain a multiresolution representation: $N-1$ resolution levels of wavelet coefficients (*HF coefficients*) and a coarsest level (*LF coefficients*). These coefficients are *tridimensional* vectors $(x_{i,1}\ x_{i,2}\ x_{i,3})$, where i stands for the resolution index. In our work, we choose the Loop DWT because this transform gives good visual results in 3D meshes compression [1]. Then, we use an optimal nearly uniform scalar quantizer with non uniform quantization step described in [8]. The quantized wavelet coefficients are entropy coded using EBCOT coder [9, 10]. This lossless context based coder, included in JPEG 2000 [2], creates an embedded bitstream. Zerotree coding as SPIHT [11] could also be a good candidate. However, EBCOT coder has been shown more efficient for images than SPIHT [2]. Also it will be used to encode the topology.

2.2. Wavelet coefficients model

In [12] we showed that most of the normalized zero-mean cross-correlations between the coordinates of wavelet coefficients are located around zero. By this way, we propose a

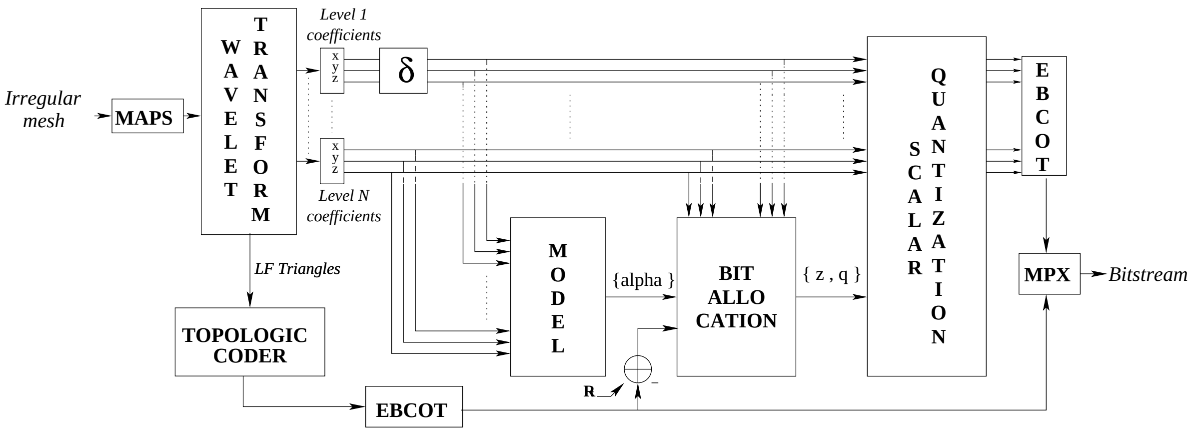


Fig. 1. Proposed compression scheme.

separate quantization process for each coordinate subband $\{x_{i,j}\}$.

2.2.1. Wavelet coefficients distribution

The only way to allocate the bitrates in the different subbands without pre-quantizing each subband is to perform a model-based bit allocation, depending on distortion and rate models and the coefficient distributions. We can observe that all HF subband distributions are zero mean and all informations are concentrated on few coefficients (very small variances) [12]. It can be shown that a good approximation for each HF coordinate pdf is given by the *generalized Gaussian distribution* [13]:

$$p(x) = ae^{-|bx|^{\alpha}} \quad (1)$$

with $b = \frac{1}{\sigma}\sqrt{\frac{\Gamma(3/\alpha)}{\Gamma(1/\alpha)}}$ and $a = \frac{b\alpha}{2\Gamma(1/\alpha)}$. The parameter α is computed using the variance and the fourth-order moment of each subband $\{x_{i,j}\}$.

2.2.2. Rate and distortion models

For each coordinate subband $\{x_{i,j}\}$, the bitrate $R_{i,j}$ related to a deadzone scalar quantizer $\{q_{i,j}, z_{i,j}\}$, is estimated by computing the entropy:

$$R_{i,j} = -\sum_{m_{i,j}=-\infty}^{+\infty} Pr(m_{i,j})\, log_2\, Pr(m_{i,j}) \quad (2)$$

with $Pr(m_{i,j})$ the probability of a quantization level $m_{i,j}$:

$$Pr(m_{i,j}) = \int_{\frac{z_{i,j}}{2}+|m_{i,j}|q_{i,j}}^{\frac{z_{i,j}}{2}+(|m_{i,j}|-1)q_{i,j}} p(x_{i,j})dx_{i,j} \quad (3)$$

$$\text{and } Pr(0) = \int_{-\frac{z_{i,j}}{2}}^{+\frac{z_{i,j}}{2}} p(x_{i,j})dx_{i,j} \quad (4)$$

Furthermore, the related model-based distortion $\sigma^2_{Q_{i,j}}$ for the i, jth subband is:

$$\sigma^2_{Q_{i,j}} = \int_{-\frac{z_{i,j}}{2}}^{+\frac{z_{i,j}}{2}} p(x_{i,j})dx_{i,j} + 2\sum_{m_{i,j}=1}^{+\infty}\int_{\frac{z_{i,j}}{2}+|m_{i,j}|q_{i,j}}^{\frac{z_{i,j}}{2}+(|m_{i,j}|-1)q_{i,j}} (x_{i,j}-\hat{x}_{i,j})^2 p(x_{i,j})dx_{i,j} \quad (5)$$

where $x_{i,j}$ is an original sample and $\hat{x}_{i,j}$ its corresponding quantized sample. For a generalized Gaussian distribution, formula (5) can be written as:

$$\sigma^2_{Q_{i,j}} = \sigma^2_{i,j} D_{i,j}\left(\frac{z_{i,j}}{\sigma_{i,j}}, \frac{q_{i,j}}{\sigma_{i,j}}\right) \quad (6)$$

with $\sigma^2_{i,j}$ the variance of subband i, j. See [8, 10] for more explanations.

3. GEOMETRY CODING

3.1. Predictive coding for LF coefficients

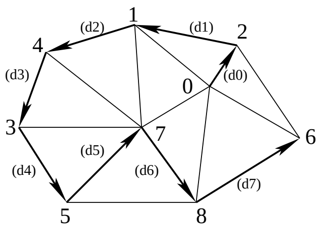
The coefficients of LF resolution level do not have any particular distribution and cannot be modeled by an unimodal function like HF ones. To overcome this problem, we use a prediction method and propose to model the *differences* between two LF coefficients instead of the LF vectors themselves. Indeed, these difference vectors can be modeled by a generalized Gaussian distribution.

Let $X_{LF} = \{X_i, for\ all\ i \in [0, \#LFvertices]\}$ be the set of LF vectors, let δ be the output set of difference vectors and I the output set of new-ordered indices. The pseudo-code is:

1. The first reference vector X is X_{init}; $I = \{init\}$; X_{LF} holds all LF vectors excepted X_0;
2. Find X_i the closest point of X among X_{LF} by minimizing $||X - X_i||^2$;

3. Add i in I and add the difference vector $(X - X_i)$ in δ;
4. Remove X_i from X_{LF}; $X \leftarrow X_i$;
5. If X_{LF} is not empty, return to step 2 else stop.

The obtained set δ represents the three *LF subbands* and will be considered by the allocation process like classical wavelet coefficients (see section 3.2). It depends on the choice of X_{init}. On the other hand, to correctly reconstruct the mesh, the set I must be known by decoder since the order of δ is different of the original list of vectors. In order to avoid an additionnal binary cost by transmitting I, we adjust at coding step the order of LF coefficients to one which is given by I (see figure 2).



Old indices	0	2	1	4	3	5	7	8	6
New indices	0	1	2	3	4	5	6	7	8

Fig. 2. Predictive coding of LF coefficients. Example with 9 vertices indexed from 0 to 8. The arrows show the predicted path found by the algorithm.

3.2. Optimal Bit allocation

3.2.1. General purpose

This is the crucial step of our compression scheme. The main idea is to determine the best set of deadzones and quantization steps $\{z_{i,j}, q_{i,j}\}$ for each subband (in our case, sets of coordinates $\{x_{i,j}\}$ for i and j fixed) that minimizes the distortion $\sigma^2_{Q_{i,j}}$ at a given rate [8].

By introducing Lagrangian operators, this constrained allocation problem can be written as:

$$J(\{z_{i,j}, q_{i,j}\}, \lambda) = \sum_{i=1}^{N}\sum_{j=1}^{3} \Delta_{i,j}\pi_{i,j}\sigma^2_{i,j} D_{i,j}(\frac{z_{i,j}}{\sigma_{i,j}}, \frac{q_{i,j}}{\sigma_{i,j}}) + \lambda(\sum_{i=1}^{N}\sum_{j=1}^{3} a_{i,j} R_{i,j}(\frac{z_{i,j}}{\sigma_{i,j}}, \frac{q_{i,j}}{\sigma_{i,j}}) - R_T) \quad (7)$$

where $\pi_{i,j}$ and $\Delta_{i,j}$ are optional weights respectively for taking account of the non-orthogonality of the filter bank and for frequency selection. The coefficients $a_{i,j}$ depend on the subsampling and correspond to $a_{i,j} = size(\{x_{i,j}\})/(3\times$ $\#$ semi-regular vertices). $D_{i,j}$ and $R_{i,j}$ depend only on α and the quotients $\frac{z_{i,j}}{\sigma_{i,j}}$ and $\frac{q_{i,j}}{\sigma_{i,j}}$.

By differentiating expression (7) with respect to $z_{i,j}$, $q_{i,j}$ and λ, and by solving the resulting system, we obtain the optimal relationships [8]:

$$h_{i,j}(\frac{q_{i,j}}{\sigma_{i,j}}) = \frac{\frac{\partial D_{i,j}}{\partial x_2}(g_i(\frac{q_{i,j}}{\sigma_{i,j}}), \frac{q_{i,j}}{\sigma_{i,j}})}{\frac{\partial R_{i,j}}{\partial x_2}(g_i(\frac{q_{i,j}}{\sigma_{i,j}}), \frac{q_{i,j}}{\sigma_{i,j}})} = -\lambda \frac{a_{i,j}}{\Delta_{i,j}\pi_{i,j}\sigma^2_{i,j}} \quad (8)$$

$$\sum_{i=1}^{N}\sum_{j=1}^{3} a_{i,j} R_{i,j}(g_{i,j}(\frac{q_{i,j}}{\sigma_{i,j}}), \frac{q_{i,j}}{\sigma_{i,j}}) - R_T = 0 \quad (9)$$

with $\frac{z_{i,j}}{\sigma_{i,j}} = g_{i,j}(\frac{z_{i,j}}{\sigma_{i,j}})$ for a given λ. $h_{i,j}$ is used in (8) to simplify the notations.
This allocation needs three functions depending on the distribution model: $ln(-h_{i,j}) = f_1(R_{i,j})$, $R_{i,j} = f_2(\frac{q_{i,j}}{\sigma_{i,j}})$ and $\frac{z_{i,j}}{\sigma_{i,j}} = g_{i,j}(\frac{q_{i,j}}{\sigma_{i,j}})$. For low complexity purposes, we use pre-computed tables.

3.2.2. Algorithm

The bit allocation algorithm is the following:

1. λ is given. Compute $-\lambda\frac{a_{i,j}}{\Delta_{i,j}\pi_{i,j}\sigma^2_{i,j}} = ln(-h_{i,j})$ and read the resulting bitrate $R_{i,j}$ from the first pre-computed tables.
2. While (9) is not verified (below a given threshold), calculate a new λ by dichotomy and return to step 1;
3. Compute $\frac{q_{i,j}}{\sigma_{i,j}}$ for each subband using the tabulated function $R_{i,j} = f_2(\frac{q_{i,j}}{\sigma_{i,j}})$;
4. Use the table $\frac{z_{i,j}}{\sigma_{i,j}} = g_{i,j}(\frac{q_{i,j}}{\sigma_{i,j}})$ to find $z_{i,j}$.

During bit allocation, the convergence is found after few iterations. Finally, subbands are quantized using the optimal set $\{z_{i,j}, q_{i,j}\}$ and encoded with EBCOT.

3.3. Experimental results

Our geometry coder is compared with the PGC method one [1]. The comparison criteria are: the bitrates (bits/vertex) with respect to the number of vertices of the semi-regular mesh and the Peak SNR: $PSNR = 20log_{10}peak/d$, with $peak$ the bounding box diagonal and d the RMSE between original semi-regular mesh and quantized one. Figures 3 show results for three 3D objects (*venus*, *rabbit* and *horse*). We can observe the efficiency of the proposed bit allocation: our results are similar or superior to those obtained by PGC method for these three objects.

4. TOPOLOGY CODING

4.1. Algorithm

To reconstruct the object, only the coarsest level triangles need to be transmitted or stored. In fact, the topology of finer resolution levels is implicit due to the subdivision connectivity remeshing [7]. By this way, we propose an efficient method to encode the coarse mesh, exploiting the new

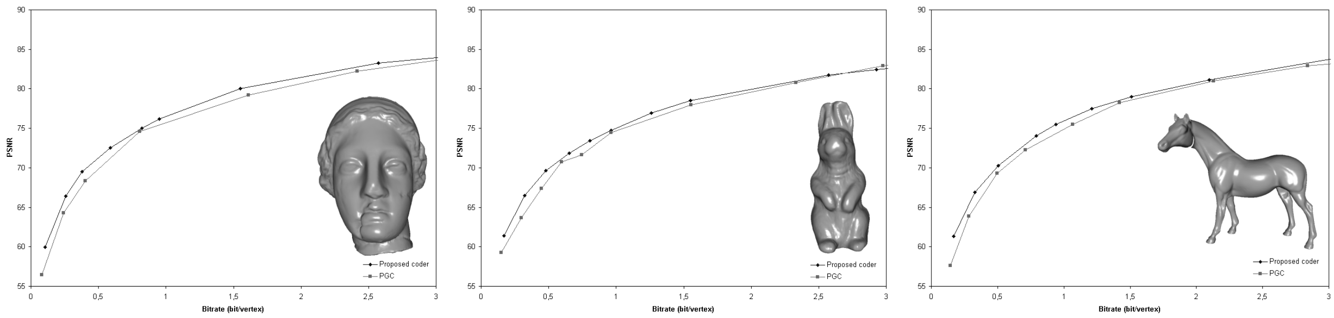


Fig. 3. Geometry coding: PSNR (on semi-regular mesh) vs Rate for venus, rabbit and horse objects.

path. Our topology coder includes two steps: reorganization of vertex indices inside triangles and edge encoding. First, the indices are renamed depending on the path and sorted in each triangle by increasing values (this permits an unique reconstruction of the topology by the decoder). Then, we introduce a right triangular matrix M to store the LF edges: $M[i, i+j]$ represents the existence of the edge (i, j). The algorithm is the following:

1. Rename vertex indices depending on the set $\{I\}$ obtained in section 3.1;
2. Sort the indices of LF triangles by increasing values;
3. Reorganize the children triangles depending on the two first steps;
4. Fill the matrix M: $M[i, i+j] = 1$ (2 for a boundary edge) if the edge (i, j) exists and 0 else;
5. Encode the values of M with EBCOT.

4.2. Experimental results

Table 1 shows the binary cost to encode the topological information. The bitrate introduced by the proposed method is negligible compared to the geometry coding cost. Moreover, it will be deduced from the target bitrate R_T to take in account the topology during bit allocation (see figure 1).

	# LF Tri.	binary cost (b)	Bitrate (b/v)
Rabbit	210	1248	$1.160E^{-2}$
Venus	388	2426	$1.221E^{-2}$
Horse	220	1272	$1.129E^{-2}$

Table 1. Topology binary cost for different objects, with respect to the number of vertices of semi-regular mesh .

5. CONCLUSIONS

In this paper, we proposed a new compression scheme using model-based geometry coding of 3D wavelet coefficients. The efficiency of this coder comes from the low complexity model-based bit allocation: bits are dispatched across subbands according to their variance. Moreover, a predictive method is used to process the LF vertices. Thus, these subbands can be modeled and introduced in the bit allocation process. It provides results slighly better than the PGC method [1]. Finally, our topology encoder permits the reconstruction of the object with a very low additionnal cost.

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