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A SYNERGIC APPROACH TO THE MINIMAL UNCOMPLETABLE WORDS PROBLEM

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ABSTRACT

A finite language X over an alphabet Σ is complete if any word in Σ^* is a factor of a word in X^*. A word which is not a factor of X^* is said uncompletable. Among them, some are minimal as all their proper factors belong to $Fact(X^*)$. The problem is to find bounds on the length of the shortest minimal uncompletable words depending on k, the maximal length of words in X. Though Restivo's conjecture stating an upper bound in $2k^2$ was already contradicted twice, the problem of the existence of a quadratic upper bound is still open. Our approach is original and synergic. We start by characterizing minimal uncompletable words. An efficient in practice algorithm is given that speeds up the search of such words. Finally, a genetic algorithm using a SAT-solver allows us to obtain new results for the first values of k.

Keywords: formal languages, automata theory, minimal uncompletable words, Restivo's conjecture

1. Introduction

Our research deals with the theory of formal languages and our interest goes to *uncompletable words*. Given a finite alphabet Σ, a set X included in Σ^* is *complete* if every word w in Σ^* is *completable* into X^*, i. e., if every w in Σ^* is a factor of some word in X^*. In the event that X is not complete, some words are uncompletable up to one letter and then are called *minimal uncompletable words*. Indeed, all proper factors of those words belong to $Fact(X^*)$.

For a given code, the equivalence between the properties of *maximal code* and of *complete set* is established in [13]. In [4], the authors studied *minimal complete sets* which are not necessarily codes. In [12], the problem about the length of the shortest minimal uncompletable words with respect to a finite set X arises. In fact, minimal uncompletable words consist in a particular instance of *minimal forbidden words* studied in [1]. In his article [12], A. Restivo conjectured that a non-complete

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