

# Fast Approximate Kernel-Based Similarity Search for Image Retrieval Task \*

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## Abstract

*In content based image retrieval, the success of any distance-based indexing scheme depends critically on the quality of the chosen distance metric. We propose in this paper a kernel-based similarity approach working on sets of vectors to represent images. We introduce a method for fast approximate similarity search in large image databases with our kernel-based similarity metric. We evaluate our algorithm on image retrieval task and show it to be accurate and faster than linear scanning.*

## 1. Introduction

In computer vision, a lot of techniques are now based on unordered sets of local features to represent the image content. In [10], Lowe shows that features based on local description like Points of Interest (PoI), provide a good representation of images.

Content-based image retrieval is usually based on a similarity function between images and provide a ranking of the database regarding a query image. Designing powerful similarities and ranking schemes on unordered sets of local features is a very challenging task. Lowe proposed a two-step technique: a fast k-Nearest Neighbors (k-NN) search for all the PoIs of the query, then a voting strategy to rank the images by counting the number of matching PoIs.

One step further has been done by K. Grauman and T. Darrell [8]: they propose to use a kernel function, the pyramid match kernel, to compute the similarity between sets of PoIs. They speed up their kernel ranking scheme using an approximate k-NN search [2]. Their scheme gives good results, but are limited to kernels that can be explicitly expressed as a dot product in an induced space<sup>1</sup>. Extensions proposed by K. Grauman

[7] as the vocabulary-guided pyramid scheme also suffers of the same limitations.

We investigate here another way to get fast scheme retrieval for kernel functions without explicit formulation in the induced space. The first requirement is to define admissible kernels working on sets of PoI vectors. We use the class of kernels on sets of vectors derived from kernels on vectors (see for instance chap. 9, [13]). These kernels have been successfully adapted for object retrieval [11, 6]. The major interest is that we are not anymore limited for the choice of the kernel.

We propose in this paper an original fast retrieval scheme, similar to the Lowe voting scheme, but using our kernel-based similarity framework. We adapt the sublinear Locality Sensitive Hashing (LSH) scheme (proposed in [9] to approximate k-NN search) to select a subset of images that should be relevant enough to be at the top of the similarity ranking. To decrease the computational complexity, the kernel is only computed on this subset. The resulting scheme is an approximation of the exact similarity ranking.

After introducing our algorithm, we evaluate the accuracy of approximation and efficiency of our approach.

## 2. Fast approximate kernel-based search

### 2.1. Kernels on bags of feature vectors

Local-based image analysis provides a powerful data representation framework. Relevant features are extracted on local patches to characterize the objects embedded in the image. Image  $I_j$  is represented by a feature bag  $B_j$  composed of  $s$  vectors  $\mathbf{b}_{sj}$ . The similarity between two bags  $B_i$  and  $B_j$  is measured by the following class of kernels on bags  $K$  [13]:

$$K(B_i, B_j) \triangleq \sum_{\mathbf{b}_{ri} \in B_i} \sum_{\mathbf{b}_{sj} \in B_j} k(\mathbf{b}_{ri}, \mathbf{b}_{sj}) \quad (1)$$

where  $k$  is a kernel function. This kernel (called minor kernel) is defined as a dot product in the space induced

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<sup>1</sup>The mapping function between the original space and the Hilbert space is explicit. This assumption limits the usable classes of kernels.

by the embedding function  $\phi$ :

$$k(\mathbf{b}_{ri}, \mathbf{b}_{sj}) = \langle \phi(\mathbf{b}_{ri}), \phi(\mathbf{b}_{sj}) \rangle \quad (2)$$

Although  $K$  kernels are quite relevant to evaluate image similarity between bags [11], they are computationally expensive. Such a kernel computation becomes intractable for image retrieval task in large databases, especially when feature bags contain many vectors. For example, about 100 to 1000 SIFT vectors are usually extracted from one image [10].

To avoid the computational complexity of this kernel while keeping its similarity power, we propose to do an approximate search: we do not compute the similarity for all the images of the database but only for a carefully chosen subset. The selection of the subset of interest is presented in the following section.

## 2.2. Approximate similarity search

Let  $I_q$  be a query image represented by a bag  $Q = \{\mathbf{q}_r\}$  containing  $r$  vectors. Denoting  $K$  the similarity, the retrieval problem in the database  $\mathcal{B}$  of bags  $B_j$  can be written as:

$$\underset{B_j \in \mathcal{B}}{\text{Sort}}(K(Q, B_j)) \quad (3)$$

In precision-oriented retrieval applications [3], the full ranking of the database is not computed, only top rank of the  $N$  images most similar to the query  $I_q$ , called TOPN, is considered (usually,  $N$  is fixed by the user). Our idea is to quickly select the images that have a *high probability* to be at the top of the ranking.

We assume that two images are *very similar* if at least two local features are similar. This local similarity is expressed using a k-NN approach: the vector  $\mathbf{b}_{sj} \in B_j$  from image  $I_j$  is similar to the vector  $\mathbf{q}_r \in Q$  from query  $I_q$  if  $\mathbf{b}_{sj} \in \text{k-NN}(\mathbf{q}_r)$ . If  $\mathcal{S}$  denotes the subset of images, from  $\mathcal{B}$ , that have at least one vector similar to one vector  $\mathbf{q}_r$  of  $Q$ :

$$\mathcal{S} = \{B_j | \exists (\mathbf{q}_r, \mathbf{b}_{sj}) \in Q \times B_j, \mathbf{b}_{sj} \in \text{k-NN}(\mathbf{q}_r)\}$$

the optimization problem in (3) is modified as follows:

$$\underset{B_j \in \mathcal{S}}{\text{Sort}}(K(Q, B_j)) \quad (4)$$

This low-restrictive definition of *very similar* images allows us to avoid missing true relevant images in  $\mathcal{S}$ . This modified optimization scheme is interesting if the computation of  $\mathcal{S}$  is fast. Instead of doing a linear scan for the k-NN search, we use an efficient indexing scheme based on LSH. Thus, the computational time to retrieve the closest neighbors will be negligible with respect to the time to compute the true kernel. The approximate search in (4) will be then about  $\frac{|\mathcal{B}|}{|\mathcal{S}|}$  faster than a brute-force linear scan in (3).

## 2.3. LSH indexing

We shortly report in this section the basic LSH functionalities to explain how we use it in our context.

LSH solves the  $(R, 1 + \epsilon)$ -NN problem: find at least one vector  $\mathbf{b}'$  in the ball  $B(\mathbf{q}, (1 + \epsilon)R)$  if there is a vector  $\mathbf{b}$  in the ball  $B(\mathbf{q}, R)$ .  $\mathbf{b} \in B(\mathbf{q}, R)$  if  $\|\mathbf{b} - \mathbf{q}\| \leq R$ . Indyk and Motwani [9] solved this problem for the Hamming metric with a complexity of  $O(n^{1/(1+\epsilon)})$  where  $n$  is the number of vectors of the database. Datar and al. [4] proposed an extension of this method which solves this problem with the Euclidian metric and with similar time performances. The method generates some hash tables of vectors, where the hashing function works on tuples of random projections of the form:  $h_{\mathbf{a},c}(\mathbf{b}) = \lfloor \frac{\mathbf{a} \cdot \mathbf{b} + c}{w} \rfloor$  where  $\mathbf{a}$  is a random vector whose each entry is chosen independently from a Gaussian distribution,  $c$  is a real number chosen uniformly in the range  $[0, w]$  and  $w$  specifies a bin width (which is set to be constant for all projections). A tuple of projections specifies a partition of the space where all vectors inside the same part have the same key. All vectors with the same key are in the same bucket  $C$ . Clearly, if the number of projections is carefully chosen, then two vectors which hash into the same bucket  $C$  will be close in the feature space. To avoid boundary effects, many hash tables are generated, each using a different tuple of projections. In practice, a proportion of these vectors (called "false matches") will be at a distance greater than  $R$  from the query vector  $\mathbf{q}$ . Thus, a check (computation of the Euclidian distance between all points  $\mathbf{b}$  of bucket  $C$  and  $\mathbf{q}$ ) is carried out to remove those "false matches".

In our case, we do not want to find just one vector  $\mathbf{b}$ , such that  $\mathbf{b} \in B(\mathbf{q}, (1 + \epsilon)R)$ , but all of them. Hence, we use a method from  $E^2\text{LSH}$  [1] which is a modified version of [4] to solve the  $(R, 1 - \delta)$ -near neighbor problem: each vector  $\mathbf{b}$  satisfying  $\|\mathbf{b} - \mathbf{q}\|_2 \leq R$  has to be found with a probability  $1 - \delta$ . Thus,  $\delta$  is the probability that a near neighbor  $\mathbf{b}$ , of query  $q$ , is not reported.

## 2.4. Fast approximate similarity search scheme

The algorithm is composed of three stages: (i) computation of buckets, (ii) selection of images, (iii) computation of the kernel on selected images.

(i) In the first stage, the hash functions are generated to split the set of all  $\mathbf{b}_{sj}$  of the database into buckets. This process is time consuming but is done off-line.

(ii) For a query  $Q = \{\mathbf{q}_r\}$ , the set of keys are computed for each vector  $\mathbf{q}_r$  with the hash functions generated during the off-line stage (i). Each key allows to quickly select a bucket  $C_i^r$  containing vectors  $\mathbf{b}_{sj}$  that

get a high probability to be in the ball  $B(\mathbf{q}_r, R)$ . A check is then carried out to eliminate vectors that are at a distance greater than  $R$  of  $\mathbf{q}_r$ . All remaining bags  $B_j$  containing at least one vector  $\mathbf{b}_{sj}$  in a bucket  $C_i^r$  are candidates:

$$\hat{S} = \{B_j | \exists(r, s, i) : (\mathbf{b}_{sj}, \mathbf{q}_r) \in [C_i^r \cap B(\mathbf{q}_r, R)] \times C_i^r\}$$

(iii) The similarity is computed only for these images; the approximate search is defined by:

$$\text{Sort}(K(Q, B_j))_{B_j \in \hat{S}} \quad (5)$$

As seen in section 2.3, some of the "good matches" (vectors in ball  $B(\mathbf{q}_r, R)$ ) are not reported, thus  $\hat{S} \subsetneq \mathcal{S}$ . However, the more similar  $B_j$  and  $Q$  are, the more vectors  $\mathbf{b}_{sj}$  close to vectors  $\mathbf{q}_r$  are likely to be found. Since one match is enough to select the corresponding image, we see that a close target image is more likely to be selected, and  $\hat{S}$  should be a relevant estimation of  $\mathcal{S}$ .

The parameter  $R$  allows to tune the approximation of the search and the time complexity of the algorithm. With  $R$  increasing, the number of selected images ( $|\hat{S}| \approx |\mathcal{B}|$ ) increases. Thus, the approximate kernel becomes more accurate; however, computational time tends to true search one ( $\frac{|\hat{S}|}{|\mathcal{B}|} \approx 1$ ).

In the next section, we empirically evaluate the tradeoff between approximation and speed up according to  $R$ .

### 3. Experiments

In this section we show that our approximate system search is efficient in the context of a content-based image retrieval. As Gosselin in [6] and Lyu in [11] have already proved the power of kernels on bags, we consider this class of kernels and focus our results on the evaluation of our approximation: ranking deterioration and computational time reduction versus search on the whole database by linear scan.

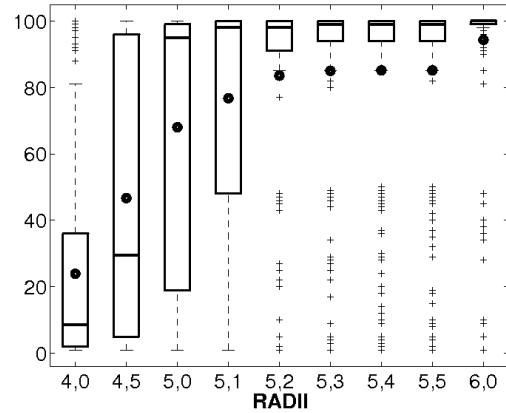
We first show that our approximation is almost equivalent to the true search, comparing the ranking of similarity by approximate search with the ranking by searching in the whole database. We focus our evaluation on the first  $N$  images of the ranking TOPN, by computing the accuracy of TOPN, defined as the number of images of the TOPN obtained with the true search actually found in the TOPN obtained with the approximate search.

Then we evaluate the computational time saved with our approximation by measuring the ratio of the number of images selected  $|\hat{S}|$  divided by the number of images in the database  $|\mathcal{B}|$ . Finally, we gave for various radii

the actual time improvement factor between the approximate and true search.

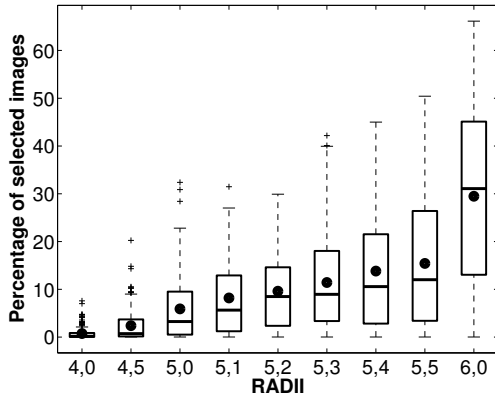
We display results of the first two evaluations with "box and whisker plots": each box has lines at the lower quartile, median value, and upper quartile values, whiskers extend from each end of the box to show the extent of the rest of data, and outliers are denoted with pulses. Circles are added to show mean values.

To evaluate our method, we used VOC2006 database [5] which contains 5,304 images. The database was indexed using well-known approach combining MSER region detectors and SIFT descriptors [12]. Each image is described by a bag of about one hundred PoI. Each PoI is described by a 128 dimensional SIFT vector representing 16 concatenated histogrammes of image gradient orientation of 8 bin each. For all our experiments, we chose as parameter of  $E^2LSH$  [1]  $\delta = 0.1$ , which corresponds to 400 hash tables of 20 projections and we tested for various radii between 4.0 and 6.0. We tested the kernel of Eq.(1) with a minor kernel Gaussian  $L_2$ . All results are obtained with 200 randomized queries. Figure (1) displays plotted results of the deterioration



**Figure 1. Accuracy of TOP100 for various radii of search around query points.**

measure induced by the approximation for  $N = 100$ . With a search radius greater than 5.2, on average, more than 80% of images of the TOP100 are reported, and for 50% of queries, it reaches 98% of accuracy. In comparison, on Caltech-101 database Grauman obtained an accuracy of 76% for a TOP5 between her hashing retrieval and a true search [8]. Although the mean accuracy quickly decreases for lower radius, some queries reach good accuracy. For example, with a radius of 5.0 and a mean accuracy of 68%, we have all the same 95% of accuracy for 50% of queries. Figure (2) displays the percentage of selected images of the database. This figure helps to explain the last result. Indeed, for a radius



**Figure 2. Percentage of selected images for various radii.**

**Table 1. Speed improvement factor regards to the true search.**

Radius	4	5	5.2	6
factor	122.17	14.85	10.03	3.19

of 5.1 and for 25% of the queries, less than 1.23% of images of the database (i.e. less than 65 images) were selected. Therefore, the accuracy could not exceed 0.65 for 25% of the queries. For some queries, we have thus selected less than 100 images which explains the bad results.

This figure also shows that for a radius of 5.2 that gives good result, on average, only 9.6% of images of the database were selected. We therefore should have an improvement of computation time of a factor of 10 towards the computation time of the true search. We have checked that the improvement of computing time was well reached by comparing obtained computation time for the true search and the approximate one (tab.1). For small radii the time improvement factor is important and it goes to one as  $R$  increases.

The average time to compute the true search for a query is about 95 seconds on a machine with a 3.2 GHz processor and 8 GB of memory. For a radius of 5.2, the selection of  $\hat{S}$  takes 0.14 second and the similarity on this selection takes 9.26 seconds. The use of LSH to select images makes the computational time of this step negligible compared to computation time of the true search, so we are  $\frac{|\hat{S}|}{|B|} \approx 10$  faster for a radius of 5.2. This approximation makes computing time acceptable for an online learning scheme.

## 4. Conclusion

In this paper, we introduced a method to efficiently achieve similarity retrieval in large image databases. Our technique is based on a powerful similarity using local-based image representation and kernel functions. We short-cut the full database linear scan by only computing the kernel-based similarity on a carefully chosen subset of images. We combined this strategy with a LSH scheme to efficiently compute the pre-selected image subset. Experiments on image datasets demonstrated that our method achieves a good tradeoff between accuracy and efficiency for the image similarity search task. We are currently working on the embedding of this strategy in online category learning scheme.

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