A generalised Golden-Section algorithm for line-search*

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Abstract—This paper aims at promoting a generalisation of the Golden-Section line-search algorithm, with better performances for functions locally symmetric around their optimum. Any line-search algorithm can be represented as a non-convergent dynamic system when a suitable renormalisation of the uncertainty interval is performed at each iteration. This allows a detailed study of the finite sample and asymptotic behaviour of the algorithm through a Markov-chain representation. We show that an expansion of the initial uncertainty interval improves this behaviour. Some asymptotic characteristics based on the evolution of the length of the uncertainty interval are shown to be related to classical ergodic characteristics of dynamic systems, such as Lyapunov exponent and Kolmogorov entropy. However, other ergodic characteristics, related to the Renyi entropy, are suggested as being more suitable in this context.

Abbreviated title—A generalised Golden-Section algorithm

Key words—Golden Section, Fibonacci, line search, dynamic system, Markov chain, entropy

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