On average-optimal quasi-symmetrical univariate optimization algorithms

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1 Introduction

This paper deals with the problem of minimizing the length of an interval containing the scalar argument x^* at which a function $f \in \mathcal{F}$ reaches its minimum value. The class $\mathcal{F}$ considered corresponds to inverse-unimodal functions over a given initial interval I_0, i.e.

$$
\begin{aligned}
& f : I_0 \rightarrow \mathcal{R} \\
& \exists x^* \in I_0 \mid f \text{ strictly decreasing for } x \leq x^* \text{ and strictly increasing for } x > x^* , \\
& \text{or else strictly decreasing for } x < x^* \text{ and strictly increasing for } x \geq x^* .
\end{aligned}
$$

The restriction to functions symmetrical at x^* will be used in Section 3, but no further assumption on f (such as regularity) will be considered. Our aim is to determine an interval I_N, after N evaluations of f in I_0, which is guaranteed to contain $x^* = \arg\min_{x \in I_0} f(x)$ and which is of minimal length. We assume that the evaluations of f are performed without errors. The result of an evaluation will be called an observation. Let $\mathcal{I}_k$ denote the information obtained after k evaluations of f at the points $x_1, x_2, \ldots, x_k$, i.e. $\mathcal{I}_k = \{a, b, x_1, x_2, \ldots, x_k, f(x_1), f(x_2), \ldots, f(x_k)\}$, with $\mathcal{I}_0 = \{a, b\}$ ($I_0 = [a, b]$). The information $\mathcal{I}_k$ is used to define I_{k+1} which is guaranteed to contain x^*, according to the rule

$$I_{k+1} = [l(\mathcal{I}_k), u(\mathcal{I}_k)] .$$