Dynamic control of Coding in Delay Tolerant Networks

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***Abstract*—We study replication mechanisms that include Reed-Solomon type codes as well as network coding in order to improve the probability of successful delivery within a given time limit. We propose an analytical approach to compute these and study the effect of coding on the performance of the network while optimizing parameters that govern routing.**

***Index Terms*—Delay Tolerant Networks, Optimal Scheduling, Coding, Network Codes**

I. Introduction

DTNs exploit random contacts between mobile nodes to allow end-to-end communication between points that do not have end-to-end connectivity at any given instant. This is obtained at the cost of replications of data and hence of energy and memory resources. To transfer successfully a file, all frames of which it is composed are needed at the destination. The memory of a DTN node is assumed to be limited to the size of a single frame. We study adding coding in order to improve the storage efficiency. We consider Reed-Solomon type codes as well as network coding. The basic questions are then: (i) **transmission policy:** When the source is in contact with a relay node, should it transmit a frame to the relay? (ii) **scheduling:** If yes, which frame should a source transfer?

Each time the source meats a relay node, it chooses a frame i for transmission with probability u^i. In a simple scenario, the source has initially all the frame and u^i are fixed in time. It was shown in [1] that the transmission policy has a threshold structure: use all opportunities to spread frame till some time σ and then stop (this is similar to the "spray and wait" policy [2]). Due to convexity arguments it turns out that the optimal u^i does not depend on i [1]. In this paper we assume a general arrival process of frames: they need not become available for transmission simultaneously at time zero as in [1]. We further consider *dynamic* scheduling: the probabilities u^i may change in time. We define various performance measures and solve various related optimization problems. Surprisingly, the transmission does not follow anymore a threshold policy (in contrast with [1]). We extend these results to include also coding, and show that all performance measures improve when increasing the amount of redundancy. We then study the optimal transmission under network coding.

II. The model

Consider a network that contains $N+1$ mobile nodes. Two nodes are able to communicate when they come within reciprocal radio range and communications are bidirectional. We assume that the duration of such contacts is sufficient to exchange all frames: this let us consider nodes *meeting times* only, i.e., time instants when a pair of not connected nodes fall within reciprocal radio range. Time between contacts of pairs of nodes are exponentially distributed with given inter-meeting intensity λ [3]. A file contains K frames. The source of the file receives the frames at some times $t_1 \le t_2 \le ... \le t_K$. t_i are called the *arrival times*. The transmitted file is relevant during some time τ. By that we mean that all frames should arrive at the destination by time $t_1+\tau$. We do not assume any feedback that allows the source or other mobiles to know whether the file has made it successfully to the destination within time τ. If at time t the source encounters a mobile which does not have any frame, it gives it frame i with probability $u_i(t)$. We assume that $u=1$ where $u=\sum_i u_i(t)$. There is an obvious constraint that $u_i(t)=0$ for $t \le t_i$. Let $\widehat{\mathbf{X}}(t)$ and $\mathbf{X}(t)$ be the n dimensional vectors whose components are $\widehat{X}_i(t)$ and $X_i(t)$. Here, $\widehat{X}_i(t)$ stand for the fraction of the mobile nodes (excluding the destination) that have at time t a copy of frame i, and $X_i(t)$ the expectation of $\widehat{X}_i(t)$.

Dynamics of the expectation. Let $X(t)=\sum_{i=1}^K X_i(t)$. The dynamics of X_i is given by

$$\dot{X}_i(t) = u_i(t)\lambda(1-X(t)) \tag{1}$$

Summing over i, we obtain $\dot{X}(t) = \lambda u(1-X(t))$ whose solution is

$$X(t) = 1 + (z-1)e^{-\lambda \int_0^t u(r)dr}, \quad X(0)=z \tag{2}$$

where z is the total initial number of frames at the system at time $t=0$. Thus, $X_i(t)$ is given by the solution of

$$\dot{X}_i(t) = -u_i(t)\lambda(z-1)e^{-\lambda \int_0^t u(r)dr} \tag{3}$$

Unless otherwise stated, we shall assume throughout $z=0$.

Performance measures and optimization.

Denote by $D(\tau)$ the probability of a successful delivery of all K frames by time τ. Define the random variable $D(\tau|\mathcal{F}_{\mathbf{X}})$ as the successful delivery probability conditioned on $\widehat{\mathbf{X}}$, where $\mathcal{F}_{\mathbf{X}}$ is the natural filtration of the process $\widehat{\mathbf{X}}$ [4]. We have

$$E[D_K(\tau|\mathcal{F}_{\mathbf{X}})] = E\left[\prod_{i=1}^{K}(1-\exp(-\lambda\widehat{Z}_i))\right] \tag{4}$$

where $\widehat{Z}_i = \int_0^\tau \widehat{X}_i(s)ds$. We shall consider the asymptotics as N becomes large yet keeping the total rate λ of contacts a constant (which means that the contact rate between any two individuals is given by $\widetilde{\lambda} = \lambda/N$). Using strong laws of large numbers, we get $\lim_{N\to\infty} \widehat{Z}_i(N) = E[\widehat{Z}_i]$ a.s. Observe that since eq. (4) is bounded, using the Dominated Convergence Theorem, we obtain

$$P_s(\tau) = \lim_{N\to\infty} E[D_K(\tau|\mathcal{F}_{\mathbf{X}}, N)] = \prod_{i=1}^{K}(1-\exp(-\lambda E[\widehat{Z}_i]))$$

Definition 2.1: u is a *work conserving* policy if whenever the source meets a node then it forwards it a frame, unless the energy constraint has already been attained.

We shall study the following optimization problems:

- **P1.** Find $\mathbf{u}$ that maximizes the probability of successful delivery till time τ.
- **P2.** Find $\mathbf{u}$ that minimizes the expected delivery time over the work conserving policies.

Definition 2.2: $\mathbf{u}$ is *uniformly optimal* for problem P1 if it is optimal for problem P1 for all $\tau > 0$.

Energy Constraints. Denote by $\mathcal{E}(t)$ the energy consumed by the whole network for transmitting and receiving a file during the time interval $[0, t]$. It is proportional to $X(t)-X(0)$ since we assume that the file is transmitted only to mobiles that do not have the file, and thus the number of transmissions of the file during $[0, t]$ plus the number of mobiles that had it at time zero equals to the number of mobiles that have it. Also, let $\varepsilon > 0$ be the energy spent to forward a frame during a contact (notice that it includes also the energy spent to receive the file at the receiver side). We thus have $\mathcal{E}(t) = \varepsilon(X(t) - X(0))$. In the following we will denote x as the maximum number of copies that can be released due to energy constraint.

Introduce the constrained problems **CP1** and **CP2** that are obtained from problems P1 and P2 by restricting to policies for which the energy consumption till time τ is bounded by some positive constant.

III. Optimal scheduling

Theorem 3.1: (**An optimal equalizing solution**) Fix $\tau > 0$. Assume that there exists some policy $\mathbf{u}$ satisfying $\sum_{i=1}^{K} u_t^i = 1$ for all t and $\int_0^\tau X_i(t)dt$ is the same for all i's. Then $\mathbf{u}$ is optimal for P1.

Not always it will be possible to equalize the above integrals. A policy $\mathbf{u}$ which is optimal among the work conservative policies will be obtained by making them as equal as possible in the majorization sense.

Theorem 3.2: $\log P_s(\tau, \mathbf{u})$ is Schur concave in $\mathbf{Z} = (Z_1, ..., Z_K)$. Hence if $\mathbf{Z}'$ majorizes $\prec \mathbf{Z}'$ then $P_s(\tau, \mathbf{u}) \geq P_s(\tau, \mathbf{u}')$.

(Majorization and Schur-Concavity are defined in [5].)

Example: The case $K = 2$. Consider the case of $K = 2$. Let the system be empty at time 0, i.e., $z = 0$, and let $t_1 = 0$. Consider the policy that transmits always frame 1 during $t \in [t_1, t_2]$, and from time t_2 onwards it transmits only frame 2. Then

$$X_1(t) = \begin{cases} \overline{X}(t) & 0 \leq t \leq t_2 \\ \overline{X}(t_2) & t_2 < t \leq \tau \end{cases}$$

where $\overline{X}(t) = 1 - \exp(-\lambda t)$. Also,

$$X_2(t) = \begin{cases} 0 & 0 \leq t \leq t_2 \\ \overline{X}(t) - X(t_2) = e^{-\lambda t_2} - e^{-\lambda t} & t_2 \leq t \leq \tau \end{cases}$$

This gives

$$\int_0^\tau X_1(t)dt = \frac{-1 + \lambda t_2 + e^{-\lambda t_2}}{\lambda} + (\tau - t_2)(1 - e^{-\lambda t_2})$$
$$\int_0^\tau X_2(t)dt = \frac{e^{-\lambda t_2}}{\lambda}(\lambda(\tau - t_2) - 1 + e^{-\lambda(\tau - t_2)})$$

We compute the value of τ for which $\int_0^\tau X_1(t)dt = \int_0^\tau X_2(t)dt$. We denote by t_{eq} the solution. We obtain (almost instantaneous with Maple 9.5)[1]:

$$t_{eq} = \frac{1}{\lambda}\left[LambertW\left(-\frac{\exp(\xi)}{1 - 2\exp(-\lambda t_2)}\right) + \xi\right]$$

$$\text{and where } \xi := \frac{-1 + 2e^{-\lambda t_2} + 2\lambda t_2 e^{-\lambda t_2}}{1 - 2e^{-\lambda t_2}}$$

Then we have the following.

Theorem 3.3: (i) Assume that $\tau < t_{eq}$. Then there is no work conserving policy that equalizes $\int_0^\tau X_1(t)dt = \int_0^\tau X_2(t)dt$. Thus there is no optimal work conserving optimal for P1. (ii) Assume that $\tau = t_{eq}$. Consider the policy $\mathbf{u}'$ that transmits always frame 1 during $t \in [t_1, t_2)$, then transmits always frame 2 during time $t \in [t_2, \tau)$. Then this work conserving policy achieves $\int_0^\tau X_1(t)dt = \int_0^\tau X_2(t)dt$ and is thus optimal for P1. (iii) Assume now $\tau > t_{eq}$. Consider the work conserving policy $\mathbf{u}^*$ that agrees with $\mathbf{u}'$ (defined in part ii) till time t_{eq} and from that time onwards uses $u_1 = u_2 = 0.5$. Then again $\int_0^\tau X_1(t)dt = \int_0^\tau X_2(t)dt$ and $\mathbf{u}^*$ is thus optimal for P1.

$\diamond$

Note that the same policy $\mathbf{u}^*$ is optimal for P1 for all horizons long enough, i.e., whenever $\tau \geq t_{eq}$ as $\mathbf{u}^*$ equalizes $\int_0^\tau X_1(t)dt = \int_0^\tau X_2(t)dt$ for all values of $\tau > t_{eq}$, because $u_1 = u_2 = 0.5$. Moreover, we have

Theorem 3.4: The work conserving policy $\mathbf{u}^*$ described at (ii) in Thm. 3.3 is uniformly optimal for problem P2.

A. Constructing an optimal work conserving policy

We propose an algorithm that has the property that it generates a policy $\mathbf{u}$ which is optimal not just for the given horizon τ but also for any horizon shorter than τ. Yet optimality here is only claimed with respect to work conserving policies.

Definitions:

- $Z_j(t) := \int_{t_1}^{t} x_j(r)dr$. We call $Z_j(t)$ the cumulative contact intensity (CCI) of class j.
- $I(t, A) := \min_{j\in A}(Z_j, Z_j > 0)$. This is the minimum non zero CCI over j in a set A at time t.
- Let $J(t, A)$ be the subset of elements of A that achieve the minimum $I(t, A)$.
- Let $S(i, A) := \sup(t : i \notin J(t, A))$.
- Define e_i to be the policy that sends at time t frame of type i with probability 1 and does not send frames of other types.

Recall that $t_1 \leq t_2 \leq ... \leq t_K$ are the arrival times of frames $1, ..., K$. Consider the Algorithm A in Table I. Algorithm A seeks to equalize the less populated frames at

[1] LambertW below is known as the inverse function of $f(w) = w\exp(w)$

TABLE I
ALGORITHM A

A1 Use $\mathbf{p}_t = e_1$ at time $t \in [t_1, t_2)$.
A2 Use $\mathbf{p}_t = e_2$ from time t_2 till $s(1,2) = \min(S(2, \{1,2\}), t_3)$. If $s(1,2) < t_3$ then switch to $\mathbf{p}_t = \frac{1}{2}(e_1 + e_2)$ till time t_3.
A3 Define $t_{K+1} = \tau$. Repeat the following for $i = 3, ..., K$:
  A3.1 Set $j = i$. Set $s(i,j) = t_i$
  A3.2 Use $\mathbf{p}_t = \frac{1}{i+1-j}\sum_{k=j}^{i} e_k$ from time $s(i,j)$ till $s(i, j-1) := \min(S(j, \{1, 2, ..., i\}), t_{i+1})$. If $j = 1$ then end.
  A3.3 If $s(i, j-1) < t_{i+1}$ then take $j = \min(j : j \in J(t, \{1, ..., i\}))$ and go to step [A3.2].

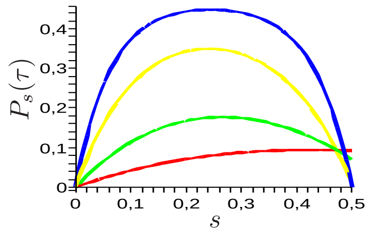

Fig. 1. Success probability under non work conserving policy $\mathbf{u}(s)$ as a function of s for $\lambda = 1, 3, 8, 15$; top curve corresponds to largest value of λ; second top corresponds to second largest λ etc. (this order changes only at s very close to 0.5).

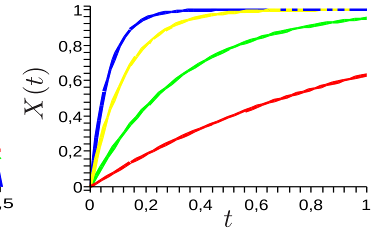

Fig. 2. The evolution of $X(t)$ as a function of t under the best work conserving policy for $\lambda = 1, 3, 8, 15$. The curves are ordered according to λ with the top curve corresponding to the largest λ etc.

each point in time: it first increases the CCI of the latest arrived frame, trying to increase it to the minimum CCI which was attained over all the frames existing before the last one arrived (step A3.2). If the minimum is reached (at some threshold s), then it next increases the fraction of all frames currently having minimum CCI, seeking now to equalize towards the second smallest CCI, sharing equally the forwarding probability among all such frames. The process is repeated until the next frame arrives: hence, the same procedure is applied over the novel interval. Notice that, by construction, the algorithm will naturally achieve equalization of the CCIs for τ large enough. Moreover, it holds the following:

Theorem 3.5: [6] Fix some τ. Let $\mathbf{u}^*$ be the policy obtained by Algorithm A when substituting there $\tau = \infty$. Then
(i) $\mathbf{u}^*$ is uniformly optimal for P2.
(ii) If in addition $\int_0^\tau X^i(t)dt$ are the same for all i's, then u^* is optimal for P1.

IV. Beyond work conserving policies

We next show the limitation of work-conserving policies.

The case K=2. We consider the example of Section III but with $\tau < t_{eq}$. Consider the policy $\mathbf{u}(s)$ where $0 = t_1 < s \le t_2$ which transmits type-1 frames during $[t_1, s)$, does not transmit anything during $[s, t_2)$ and then transmits type 2 frames after t_2. It then holds

$$X_1(t) = \begin{cases} X(t) & 0 \le t \le s \\ X(s) & s \le t \le \tau \end{cases}$$

where $X(t) = 1 - \exp(-\lambda t)$. Also,

$$X_2(t) = \begin{cases} 0 & 0 \le t \le t_2 \\ X(t - (t_2 - s)) - X(s) = & \\ e^{-\lambda s} - e^{-\lambda(t-(t_2-s))} & t_2 \le t \le \tau \end{cases}$$

This gives

$$\int_0^\tau X_1(t)dt = \frac{-1 + \lambda s + e^{-\lambda s}}{\lambda} + (\tau - s)(1 - e^{-\lambda s})$$
$$\int_0^\tau X_2(t)dt = \frac{e^{-\lambda s}}{\lambda}(\lambda(\tau - t_2) - 1 + e^{-\lambda(\tau - t_2)})$$

Example 4.1: Using the above dynamics, we can illustrate the improvement that non work conserving policies can bring. We took $\tau = 1$, $t_1 = 0$, $t_2 = 0.8$. We vary s between 0 and t_2 and compute the probability of successful delivery for $\lambda = 1, 3, 8$ and 15. The corresponding optimal policies $u(s)$ are given by the thresholds $s = 0.242, 0.242, 0.265, 0.425$. The probability of successful delivery under the threshold policies $u(s)$ are depicted in Figure 1 as a function of s which is varied between 0 and t_2.

In all these examples, there is no optimal policy among those that are work conserving. A work conserving policy turns out to be optimal for all $\lambda \le 0.9925$.

Note that under any work conserving policy, $\int_0^\tau X_2(t)dt \le \tau(1 - X(t_2))$ (where $X(t_2)$ is the same for all work conserving policies). Now, as λ increases to infinity, $X(t_2)$ and hence $X_1(t_2)$ increase to one. Thus $\int_0^\tau X_2(t)$ tends to zero. We conclude that the success delivery probability tends to zero, uniformly under any work conserving policy.

Recall that Theorem 3.3 provided the globally optimal policies for $t_{eq} \le \tau$ for $K = 2$. The next Theorem completes the derivation of optimal policies for $K = 2$ by considering $t_{eq} > \tau$.

Theorem 4.1: [6] For $K = 2$ with $t_{eq} > \tau$, there is an optimal non work-conserving threshold policy $\mathbf{u}^*(s)$ whose structure is given in the beginning of this subsection. The optimal threshold is given by $s = \frac{1}{\lambda}\log\left(1 - e^{-\lambda(\tau - t_2)}\right)$. Any other policy that differs from the above on a set of positive measure is not optimal.

A. Time changes and policy improvement

Lemma 4.1: Let $p < 1$ be some positive constant. For any multi-policy $\mathbf{u} = \{u_1(t), ..., u_n(t)\}$ satisfying $u = \sum_{i=1}^n u_i(t) \le p$ for all t, define the policy $\mathbf{v} = \{v_1, ..., v_n\}$ where $v_i = u_i(t/p)/p$ or equivalently, $u_i = pv_i(tp)$, $i = 1, ..., n$. Define by X_i the state trajectories under $\mathbf{u}$, and let $\overline{X}_i$ be the state trajectories under $\mathbf{v}$. Then $X(t) = \overline{X}(tp)$.

The control $\mathbf{v}$ in the Lemma above is said to be an *accelerated* version of $\mathbf{u}$ from time zero with an accelerating factor of $1/p$. An acceleration $\mathbf{v}$ of $\mathbf{u}$ from a given time t' is defined similarly as $v_i(t) = u_i(t)$ for $t \le t'$ and $v_i(t) = u_i(t' + (t - t')/p)/p$ otherwise, for all $i = 1, ..., n$.

TABLE II
ALGORITHM B

B1	Use $\mathbf{p}_t = u_t e_1$ at time $t \in [t_1, t_2)$.
B2	Use $\mathbf{p}_t = u_t e_2$ from time t_2. till $\min(S(2,\{1,2\}), t_3)$. If $S(2,\{1,2\}) < t_3$ then switch to $\mathbf{p}_t = \frac{1}{2}(e_1+e_2)u_t$ till time t_3.
B3	Define $t_{K+1} = \tau$. Repeat the following for $i = 3, ..., K$:
B3.1	Set $j = i$. Set $s(i,j) = t_i$
B3.2	Use $\mathbf{p}_t = \frac{1}{i+1-j}\sum_{k=j}^{i} e_k u_t$ from time $s(i,j)$ till $s(i,j-1) := \min(S(j,\{1,2,...,i\}), t_{i+1})$. If $j = 1$ then end.
B3.3	If $s(i,j-1) < t_{i+1}$ then take $j = \min(j : j \in J(t,\{1,...,i\}))$ and go to step [B3.2].

We now introduce the following policy improvement procedure.

Definition 4.1: Consider some policy $\mathbf{u}$. and let $u := \sum_{j=1}^{n} u_j(t)$. Assume that $u \leq p$ over some $0 < p < 1$ for all t in some interval $S = [a,b]$ and that $\int_b^c u(t)dt > 0$ for some $c > b$. Let w be the policy obtained from u by
(i) accelerating it at time b by a factor of $1/p$,
(ii) from time $d := a + p(b-a)$ till time $c - (1-p)(b-a)$, use $w(t) = u(t+b-d)$. Then use $w(t) = 0$ till time c.

Let $X(t)$ be the state process under u, and let $\overline{X}(t)$ be the state process under w. Then

Lemma 4.2: Consider the above policy improvement of u by w. Then (a) $\overline{X}_i(t) \geq X_i(t)$ for all $0 \leq t \leq c$, (b) $X_i(c) = \overline{X}_i(c)$ for all i, (c) $\int_a^c X_i(t)dt \leq \int_a^c \overline{X}_i(t)dt$.

B. Optimal policies for $K > 2$.

Theorem 4.2: Let $K > 2$. Then an optimal policy exists with the following structure:

- (i) There are thresholds, $s_i \in [t_i, t_{i+1}]$, $i = 1, ..., K$. During the intervals $[s_i, t_{i+1})$ no frames are transmitted.
- (ii) Algorithm B to decide what frame is transmitted at the remaining times.
- (iii) After time t_K it is optimal to always transmit a frame. An optimal policy u satisfies $u(t) = 1$ for all $t \geq t_K$ (it may differ from that only up to a set of measure zero).

V. THE CONSTRAINED PROBLEM

Let $\mathbf{u}$ be any policy that achieves the constraint $\mathcal{E}(\tau) = \varepsilon x$ as defined in Section II. We make the following observation. The constraint involves only $X(t)$. It thus depends on the individual $X_i(t)$'s only through their sum; the sum $X(t)$, in turn, depends on the policies u_i's only through their sum $u = \sum_{i=1}^{K} u_i$.

Work conserving policies. Any policy which is not a threshold one can be strictly improved as described in Lemma 4.2. Consider the case of work conserving policies. Then the optimal policy is of a threshold type [7]: $u = 1$ till some time s and is then zero. s is the solution of $X(s) = z + x$, i.e.

$$s = -\frac{1}{\lambda}\log\left(\frac{1-x-z}{1-z}\right),$$

Algorithm A can be used to generate the optimal policy components $u_i(t)$, $i = 1, \ldots, K$.

General policies Any policy $\mathbf{u}$ that is not of the form as described by (i)-(ii) in Theorem 4.2 can be strictly improved by using Lemma 4.2. Thus the structure of the optimal policies is the same, except that (iii) of Theorem 4.2 need not to hold.

VI. ADDING FIXED AMOUNT OF REDUNDANCY

We now consider adding forward error correction: we add H redundant frames and consider the new file that now contains $K + H$ frames. Under an erasure coding model, we assume that receiving K frames out of the $K + H$ sent ones permits successful decoding of the entire file at the receiver.

Let $S_{n,p}$ be a binomially distributed r.v. with parameters n and p, i.e., $P(S_{n,p} = m) = B(p,n,m) := \binom{n}{m}p^m(1-p)^{n-m}$ The probability of successful delivery of the file by time τ is thus

$$P_s(\tau, K, H) = \sum_{j=K}^{K+H} B(D_i(\tau), K+H, j),$$

where $D_i(\tau) = 1 - \exp(-\lambda\int_0^\tau X_i(s)ds)$ is the probability that frame i is successfully received by the deadline.
We assume below that the source has frame i available at time t_i where $i = 1, ..., K+H$. In particular, t_i may correspond to the arrival time of the original frames $i = 1, ..., K$ at the source. For the redundant frames, t_i may correspond either to (i) the time at which the redundant frames are created by the source, or to (ii) the moments at which they arrive at the source in the case that the coding is done at a previous stage.

Main Result

Let $Z_i = \int_0^\tau X_i(v)dv$, where $i = 1, 2, ..., K+H$.

Theorem 6.1: (i) Assume that there exists some policy $\mathbf{u}$ such that $\sum_{i=1}^{K+H} u_i(t) = 1$ for all t, and such that Z_i is the same for all $i = 1, ..., K+H$ under $\mathbf{u}$. Then $\mathbf{u}$ is optimal for P2.
(ii) Algorithm A, with $K+H$ replacing K, produces a policy which is optimal for P2.

Remark 6.1: If the source is the one that creates the redundant frames, then we assume that it creates them after t_K. However, it could use less than all the K original frames to create some of the redundant frames and in that case, redundant frames can be available earlier. E.g., shortly after t_2 it could create the xor of frame 1 and 2. We did not consider this coding policy and such option will be explored in the following sections.

In the same way, the other results that we had for the case of no redundancy can be obtained here as well (those for P1, CP1 and CP2).

VII. RATELESS CODES

We want to quantify the gains brought by rateless coding for our problem. In the reminder, information frames are the K frames received at the source at $t_1 \leq t_2 \leq \cdots \leq t_K$. The encoding frames (also called coded frames) are linear combinations of some information frames, and will be created according to the chosen coding scheme.

TABLE III
ALGORITHM C

C1	to C3 A1 to A3
C4	From $t = t_K$ to $t = \tau$, use all transmission opportunities to send a random linear combination of information frames, with coefficients picked uniformly at random in $\mathbb{F}_q$.

As in the previous section, we assume that redundant frames are created only after t_K, i.e., when all information frames are available. The case when coding is started before receiving all information frames is postponed to the next section. For a discussion on the different rateless codes for both cases, the reader is refered to [6]. In this section we provide the analysis of the optimal control with random linear network coding [8]. Note that, in our case, the coding is performed only by the source since the relay nodes cannot store more than one frame. For each generated encoding frame, the coefficients are chosen uniformly at random for each information frame, in the finite field of order q, $\mathbb{F}_q$. The decoding of the K information frames is possible at the destination if and only if the matrix made of the headers of received frames has rank K.

Recall the definition $Z_i = \int_0^\tau X_i(v)dv$, $i = 1, \dots, K-1$.

Theorem 7.1: Let us consider the above rateless coding scheme for coding after t_K.
(i) Assume that there exists some policy $\mathbf{u}$ such that $\sum_{i=1}^{K-1} u_i(t) = 1$ for all t, and such that Z_i is the same for all $i = 1, \dots, K-1$ under $\mathbf{u}$. Then $\mathbf{u}$ is optimal for P2. (ii) Algorithm C produces a policy which is optimal for P2.

VIII. Rateless codes for coding before t_K

We now consider the case where after receiving frame i and before receiving frame $i+1$ at the source, we allow to code over the available information frames and to send resulting encoding frames between t_i and t_{i+1}. We present how to use network codes in such a setting. The objective is the successful delivery of the entire file (the K information frames) by time τ[2]. Information frames are not sent anymore, only encoding frames are sent instead. At each transmission opportunity, an encoding frame is generated and sent with probability $u(t)$.

Theorem 8.1: (i) Given any forwarding policy $u(t)$, it is optimal, for maximizing $P_s(\tau)$, to send coded frames resulting from random linear combinations of all the information frames available at the time of the transmission opportunity.
(ii) For a constant policy $u > 0$, the probability of successful delivery of the entire file is lower-bounded by

$$P_s(\tau) \geq \sum_{j=0}^{K-1} \sum_{k_1 > \dots > k_j} \sum_{l_0 = K-k_1}^{K} \cdots \sum_{l_j = K - \sum_{i=0}^{j-1} l_i}^{k_j} \prod_{i=0}^{j} f(l_i, k_i) \ ,$$

with $f(l,k) = \begin{cases} P_{l,k,l} D_{k,l}(\tau), & \text{if } l < k, \\ P_{k,k,k} \left(1 - \sum_{m=0}^{k-1} D_{k,m}(\tau)\right), & \text{if } l = k \end{cases}$

and $P_{l,k,l} = \prod_{r=0}^{l-1} \left(1 - \frac{1}{q^{k-r}}\right)$, $D_{k,i}(\tau) = \exp(-\Lambda_k) \frac{\Lambda_k^i}{i!}$, and

$$\Lambda_K = \lambda \left[\exp(-\lambda u t_K) \left(\tau - t_K - \frac{1}{\lambda u} \right) + \frac{1}{\lambda u} \exp(-\lambda u \tau) \right] \ .$$

Let us briefly compare the successful delivery probabilities for the different coding schemes: Coding with rateless codes after t_K allows to need an equalization of the Z_i only for $i = 1, \dots, K-1$, i.e., for the information frames but not for the coded frames, unlike the scheme with fixed amount of redundancy. Coding before t_K avoids the need for any policy $\mathbf{u}$ for each frame in order to equalize the Z_i. This is due to the fact that, when transmitting a single coded frame, network coding allows to propagate an equivalent amount of information of each information frame, thereby circumventing the coupon collector problem that would emerge with single repetition of frames. Algorithm A addresses this problem by striving to equalize the Z_i. Hence, even though all the frames over $E(k_i)$ do not reach the destination, it is sufficient to receive more frames over $E(k_j)$, $j > i$, to recover the file.

IX. Conclusions

In this paper we addressed the problem of optimal transmission policies in two hops DTN networks under memory and energy constraints. We tackled the fundamental scheduling problem that arises when several frames that compose the same file are available at the source at different time instants. The problem is then how to optimally schedule and control the forwarding of such frames in order to maximize the delivery probability of the entire file to the destination. We solved this problem both for work conserving and non work conserving policies, deriving in particular the structure of the general optimal forwarding control that applies at the source node. Furthermore, we extended the theory to the case of fixed rate systematic erasure codes and network coding. Our model includes both the case when coding is performed after all the frames are available at the source, and also the important case of network coding, that allows for dynamic runtime coding of frames as soon as they become available at the source.

[2] We do not have constraints on making available at the destination a part of the K frames in case the entire file cannot be delivered.

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