Inter-session Network Coding in Delay-Tolerant Networks Under Spray-and-Wait Routing

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***Abstract*—The goal of this paper is to derive the performance analysis for a unicast session, under Spray-and-Wait routing and inter-session network coding, in the presence of background traffic in DTN with homogeneous mobility. General settings are considered for the distribution of the data rate between nodes, the buffer size, the number of sessions contending for network resources. Thanks to mean-field approximations, fluid models are derived to predict the dissemination of the packets pertaining to different flows, thereby presenting the first, to the best of the authors knowledge, study of performance in DTN with Spray-and-Wait under the presence of contention between several sessions. The accuracy of the model is assessed through simulations based on synthetic mobility traces.**

I. Introduction

Mobile Ad hoc Networks (MANET) aim at allowing communication between mobile users without any infrastructure. If the spatial node density of a MANET is low, then end-to-end communication between a source and a destination is limited by the lack of connectivity. That is, in order to exchange packets, two mobile nodes must come into the radio range of each other. Owing to the intermittent connectivity, the nodes must rely on the Store-Carry-and-Forward paradigm which inherently entails a delay for communication. That is the reason why such sparse MANET are referred to as Delay Tolerant Networks (DTN). In this work, we consider DTN characterized by exponentially distributed inter-contact times. Hereafter, a DTN with identically (resp. non-identically) distributed inter-contact times across the nodes is referred to an "homogeneous" (resp. "heterogeneous") DTN. Thus, in order to decrease the transmission delay, a source of traffic has to rely on the mobility of other nodes which act as relays, and takes advantage of the transmission opportunities which occur when the mobile relays come into contact. This forwarding strategy is known as opportunistic routing.

Specifically, the packet delay will be lowered if multiple copies of the same packet are allowed to spread in the network. How to spread multiple copies has been investigated in several proposals. Epidemic routing has been proposed [1] to flood data packets to all nodes in the network, sparing the maximum of energy to minimize the delay. However, most mobile nodes in DTNs have limited energy and may prefer fewer transmissions to prolong network lifetime. For these reasons, Spray-and-Wait [2] and probabilistic routing [3] are proposed to achieve trade-offs between network resource consumption and protocol performance.

On the other hand, random network coding [4] has attracted an increasing interest for DTNs [5], [6]. The benefits are increase in throughput, as well as adaptability to network topology changes and resilience to link failures. The successful reception of information does not depend on receiving a specific packet, but on receiving a sufficient number of independent packets, thereby circumventing the coupon collector problem that would emerge with single repetition of packets. There are two types of NC: intra-session NC codes together only the packets belonging to the same session or connection, while inter-session NC codes separate sessions packets. Inter-session NC is necessary to achieve optimal throughput in general (see [7] and references therein).

The goal of this paper is to derive the analysis of a network-coded transmission, under Spray-and-Wait routing, in the presence of background traffic in DTN with homogeneous mobility. The motivation to carry out such an analysis is to establish reference analysis for transmission using network coding in DTN in the homogeneous case, so as to then get allowed to go one step further towards the analysis of dissemination in DTN with heterogeneous mobility, the so-called mobile social networks. In such networks, we will, in an upcoming paper, consider the problem of dissemination when several unicast sessions are established between nodes pertaining to different social group (i.e., with different inter-contact features). Specifically, the present analysis will help devise new inter-session coding schemes to enhance performance.

Contributions

The contributions of this paper are twofold:

- We carry out the analysis of a network-coded transmission in a DTN in the presence of background traffic. Specifically, we are able to derive the distribution of the number of decodable packets received at the destination within a certain time, and hence the distribution of the delay. The analysis is carried out through mean-field approximations leading to a fluid model of the network state. Quantities of interest are then derived with Ordinary Differential Equations (ODE).
- We model dissemination with Spray-and-Wait using fluid models. We make no specific assumptions for the distri-

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bution of the data rate between the nodes and the buffer sizes.

Related works

Intra-session network coding has been well studied and understood for DTN. Papers [8] and [9] propose a technique to erasure code a file and distribute the generated code-blocks over a large number of relays in DTNs. The use of erasure codes is meant to increase the efficiency of DTNs under uncertain mobility patterns. In [9] the performance gain of the coding scheme is compared to simple replication, i.e., when additional copies of the same file are released. The benefit of erasure coding is shown by means of extensive simulations and for different routing protocols, including two-hop routing. In [10] ODE-based models are employed under epidemic routing; in that work, semi-analytical and numerical results are reported describing the effect of finite buffers and contact times over a unicast network-coded session. Specifically, the assumption is made that, owing to uniform mobility model, the rank of any number of received coded packets at the destination is the maximum possible. The same authors in [6] investigate the use of network coding using the Spray-and-Wait algorithm and analyze the performance in terms of the bandwidth of contacts, the energy constraint and the buffer size. However, no background traffic is assumed, i.e., the unicast session of interest is assumed to be the only one running in the network. In this work, we lift such assumption and derive a fluid model for information dissemination allowing to account for contending sessions. Also, our analysis accounts for arbitrary bandwidth and buffer size. In [11], Diana and Lochin propose a Markov chain-based model to obtain the end-to-end delay homogeneous and heterogeneous DTN under Spray-and-Wait routing. As the number of states of the Markov chain depends on the maximum number of copies L (203 states for $L = 32$), the computational complexity will prevent from using it for large networks requiring larger values of L. Also, the spreading of a packet is tracked assuming that the network is empty of any other packet, and hence it accounts neither for the simultaneous dissemination of many packets belonging to the same message, nor for the case of contending sessions. Network coding is therefore not considered. A REVOIR: To the best of our knowledge, no work has yet investigated the performance of inter-session network coding in DTN, nor the modeling of several contending network-coded transmission under the general Spray-and-Wait algorithm.

The paper is organized as follows. In Section II we describe the network model and discuss the existing models of network-coded transmissions in DTN. In Section III, we derive the performance analysis of two contending flows, i.e., the performance of a network-coded session in the presence of background traffic. Section IV gives simulation results assessing the accuracy of the derived performance analysis. Section VI concludes the paper.

II. Network model

In this paper, we consider a unicast communication from a source S to a destination D in a DTN with N wireless nodes, moving within a closed area. This unicast session is referred to as *the session of interest* from now onward, as opposed to all the other flows. The network is assumed sparse: the ratio between the coverage area of all nodes and the total area is low enough so that we neglect interference. We assume that two nodes are able to communicate when they are within reciprocal radio range, and that communications are bidirectional. Furthermore, let the time between two consecutive contacts be exponentially distributed with a certain mean. The meeting intensity is defined as the inverse of this mean. The mean number of contacts per time unit between a given pair of nodes (resp. a given node and any other node) is called intra-meeting (resp. inter-meeting) intensity and is denoted by β (resp. λ). The sparsity of network translates by keeping λ fixed in case N increases (in case β is fixed, then the network gets denser with N). The validity of this model has been discussed in [12], and its accuracy has been shown for a number of mobility models (Random Walker, Random Direction, Random Waypoint). We assume that the file to be transferred needs to be split into K packets: this occurs owing to the finite duration of contacts among mobile nodes or when the file is large with respect to the buffering capabilities of nodes. The message is considered to be well received if and only if all the K packets of the source are recovered at the destination. We assume that the transmitted message, made of the K packets of the source, is relevant during some time τ. We do not assume any feedback which allows the source or other mobiles to know whether the message has made it successfully to the destination within time τ. Contrary to previous work on the topic, we do assume that there is background traffic beyond the unicast communication of interest.

We assume that the bandwidth, defined as the number of packets that can be exchanged during a contact, is stochastic and follows a known distribution. The buffer size is assumed to be any known integer, denoted by B, equal for all nodes in the network.

III. Network-coded transmission protocols and analysis

A. Inter-session network coding

All nodes can identify the session number of each packet. We shall consider that nodes can perform inter-session NC. Consider that packets P_1 and P_2, belonging to sessions S_1 and S_2, are RLCs of K_1 and K_2 source packets, respectively. Header coefficients of P_1 and P_2 are hence K_1-long and K_2-long, while payloads are L_1 and L_2-long, where L_1 and L_2 are the maximum size of packets of S_1 and S_2, respectively. The packet resulting from RLC of P_1 and P_2 has header coefficients $(K_1 + K_2)$-long, and payload $\max(L_1, L_2)$-long. To retrieve the K_1 packets of interest, the destination has to receive the vector space sent by the source of S_1 of dimension equal to K_1. As different labels are assigned to all packets sent by the sources of the different sessions, then packet mixing at relay nodes can be performed over all the packets seemingly. Thereafter, a source packet involved in an

RLC is called a degree of freedom (DoF), and an RLC is said to be innovative if it increases the header' rank of RLC already present at the receiver. The better share of network resources provided by inter-session network coding make it appealing for applications in DT-MANETs, especially when mixing the flows destined for different nodes pertaining to the same geographical cluster can be beneficial.

B. Analysis of NC in DTNs

Let us first briefly review the possible ways of applying and analyzing NC in DTN with opportunistic routing.

When two-hop routing is used, i.e., relay nodes are not allowed to themselves relay the packets they carry from other nodes, but only to forward them directly from the source to the destination, there is no intermediate coding opportunity. Hence, coding is only performed at the source. The the source only sends random linear combination (RLC) of its K information packets, and there is no gain to code at the relay nodes. To retrieve the K information packets with a high probability which depends only on the finite field order q and K, it is sufficient for the destination to collect any K RLCs. Hence it is sufficient to track the total number of packets in the network so as to derive the success rate for reception of the K information packets within a given delay τ.

In [10], NC-based epidemic routing is presented. Packets are flooded into the network, without restriction, except that of the buffer size and the contact duration-rate product. All the packets in the buffer of a relay node are coded together to create a new packet which will be given by the relay to another relay. An upper-bound on the performance is obtained by tracking the number of nodes with every buffer states [6], under the assumption that every meeting brings innovative information.

Binary Spray-and-Wait routing (SaW) [2] intends to limit the number of copies of a given packet to M. The node buffer is structured with a spray list of the indexes of packets, each index associated with a spray-counter and the packet itself. When two nodes meet, they exchange the packets whose indexes are not in common, provided those indexes have spray-counter strictly greater than one. If the contact bandwidth allows to exchange only a limited number of packets, then those with highest spray-counters are selected. When a packet is replicated, the sending nodes keeps a copy and updates the spray-counter from x to $\lceil x/2 \rceil$ while that in the receiving node is set to $\lfloor x/2 \rfloor$. For NC-based Binary Spray-and-Wait [13], the source sends K' RLCs, each labeled with indexes from 1 to K' (with $K' \geq K$). When a node sends say p packets to another node, it sends p random linear combinations of all the packets currently in its buffer, choosing the p labels of each RLCs among those present in the buffer and with minimum spray-counters. In this case, two nodes with the same spray list do not exchange any packet, whereas they may have innovative packets for each other: the labels represent only a subset of the source packets involved in the RLCs present in the node buffer, and possibly innovative transmissions are therefore skipped in order to limit spreading. Compared with replication-based SaW, NC-based SaW propagates indexes the same way, but RLCs stored at relays are combinations of all DoFs met at the successive relay nodes. However, in the relay-to-destination phase, as a node is allowed to forward RLCs to the destination even whatever the spray-list and spray-counters are, the fact that the spray list is not representative of all the DoFs involved in the RLCs present in the node buffer is accounted for, and the rank of the destination can still increase. Such transmission protocol can be analyzed in the same way as a replication protocol, by tracking the spreading of each packet index. If the analysis of this protocol is performed with $M = N$, then it allows to get a lower-bound on the performance of NC-based epidemic routing. Contrary to previous works [6], we develop analysis of NC-based SaW routing explicitly accounting for arbitrary distribution of the contact bandwidth and the buffer size, as well as for background traffic.

IV. SPREADING ANALYSIS

We consider spray-and-wait for network-coded transmissions of several sessions. For the sake of space, we only give a sketch of the formal proof which allows us to consider the limit behavior of the system when N tends to infinity, so as to get fluid model of the performance. Considering the (set of) random variable(s) coding for the state of the network, we can express the drift driving the expected evolution of such quantities which are branching processes. We can show (see below the ODE) that they are interaction processes with vanishing intensity, owing to the dependence of the drift on β, and hence on N. We then cannot readily apply the convergence results of [14]. In such cases, Benaïm and Le Boudec have shown in [15] that, provided that we change the time scale, the re-scaled process converges for large N, in mean-square, to a deterministic dynamical system which is the solution of a certain ODEs expressed below.

The source and the destination are denoted by S and D. The set of nodes generating other sessions than the session of interest is denoted by A. Let $X_i(t)$ (resp. $Y_i(t)$) be the number of nodes having i indexes from S (resp. A) in their buffers, for $i = 1, \ldots, B$. An index is attached to an RLC (coded packet) sent from the sources of all sessions. K' is the total number of indexes sent by S. We assume the number of indexes sent by other sessions is not limited. Let M and Q be the maximum spray-counters for the sessions of S and A, respectively. We present thereafter the performance modeling for spray-and-wait when packet exchange is not possible once the buffer of the receiving node is full, but note that it is straightforward to adapt the framework to two-hop routing or for exchange when the buffers are full. We also assume that not all nodes of the network may accept to relay RLCs combining some packets A: N_s denotes the number of nodes (thereafter referred to as *secure*) not relaying other sessions than that of interest. We always assume that D is not a secure node. Note that the considered framework also fits to an adversarial environment. Let $\tilde{X}_I$ (resp. $\tilde{Y}_I$) be the number of copies of index I from S (resp. A).

A. Per buffer occupancy tracking

The ODE equations for $X_i(t)$ and $Y_i(t)$ write as:

$$dX_i/dt = \beta N \sum_{j=0}^{i-1} \sum_{l=0}^{B-i} P_{gs}(i-j, j+l) P_j(j,l) \dots$$

$$+\beta N \sum_{j=i+1}^{B} \sum_{m=0}^{B-j} P_{ls}(j-i, j, m) - \beta N \sum_{j=0}^{B-i-1} P_j(i,j) P_{gsv}(i,j) \dots$$

$$-\beta N \sum_{j=0}^{B-i} P_{lsv}(i,j) + \beta N N \sum_{j=0}^{i-1} \sum_{k=i-j}^{B} P_{grs}(i-j, k, j) \dots$$

$$-\beta N N \sum_{k=1}^{B} P_{grsv}(k,i)$$

$$dY_i/dt = \beta N \sum_{j=0}^{i-1} \sum_{l=0}^{B-i} P_{ga}(i-j, j+l) P_j(l,j) \dots$$

$$((j>0) + (j==0)P_{cgi0} + (j==0)P_{ucus}) \dots$$

$$+\beta N \sum_{j=i+1}^{B} \sum_{m=0}^{B-j} P_{la}(j-i, j, m) - \beta N \sum_{j=0}^{B-i-1} P_j(j,i) P_{gav}(j,i) \dots$$

$$-\beta N \sum_{j=0}^{B-i} P_{lav}(j,i) + \beta N N \sum_{j=0}^{i-1} \sum_{k=i-j}^{B} P_{gra}(i-j, k, j) \dots$$

$$-\beta N N \sum_{k=1}^{B} P_{grav}(k,i)$$

- $\mathbf{P_j}$: P_j is a matrix of size $B \times B$. $P_j(i,j)$ is the joint probability that a node has in its buffer both i indexes from S and j indexes A. Such a node is denoted an (i,j)-node hereafter. Then the elements of P_j are found so as to fulfil the following constraints: $(i) P(i,j) = 0$, for $j > B-i$, $(ii)$for $i = 0, \dots, B, \sum_{j=0}^{B-i} P(i,j) = X_i/N$, (iii) for $j = 0, \dots, B, \sum_{i=0}^{B-i} P(i,j) = Y_i/N$. Those equations ensure that, for all $i = 0, \dots, B$, $X_i \le \sum_{j=0}^{B-i} Y_j$ and $Y_i \le \sum_{j=0}^{B-i} X_j$.
- $\mathbf{P_{nic}}$: Given two nodes, N_1 with b_{sf} and N_2 with b_{sr} indexes of S (or A) in their buffers, $P_{nic}(k, b_{sf}, b_{sr}, K(t), K_U(t), v(t))$ gives the probability that there are k indexes of S (or A) at N_1 which are not in common with those at N_2 and whose corresponding spray-counters are still below M (or Q), when S (or A) has already spread out $K(t)$ indexes. $K_U(t)$ denotes the number of indexes spread up to t which have still spray-counter lower than M (or Q). The occurrence probabilities of every indexes of S (resp. A) at time t are stored into vector $v(t) = [\frac{\tilde{X}_1}{\tilde{X}}, \dots, \frac{\tilde{X}_{K(t)}}{\tilde{X}}]$ (resp. $w(t)$), with $\tilde{X} = \sum_I \tilde{X}_I$. Let $T = \{I : \tilde{X}_I > 0\}$ and E be the set of indexes of S that can still spread: $E = \{I : 0 < \tilde{X}_I < M\}$. Let S be a set of pairwise different elements from $T = \{1, \dots, K(t)\}$, whose cardinal is $|S|$. Then $Sz(K(t), z, v) = \sum_{S \in T : |S| = z} \prod_{i \in S} v_i \prod_{i \in T-S} (1 - v_i)$. Computing the probability that two nodes exchange packets in such a way allows to account for the non-uniform distribution of number of copies over all the sent indexes. Let us then define the probabilities that the number of indexes not in common be greater than a certain k:

$P_{cn}(k, b_{sf}, b_{sr}, K(t), K_U(t), v(t)) =$

$$\begin{cases} 1 & \text{, if } k < b_{sf} - b_{sr} \\ 0 & \text{, else if } k > b_{sf} \\ \dfrac{\binom{b_{sf}}{k}(b_{sr}+k)! Sz(K(t), b_{sr}+k, v(t))}{\sum_{k=0}^{b_{sf}} \binom{b_{sf}}{k}(b_{sr}+k)! Sz(K(t), b_{sr}+k, v(t))} & \text{, else if } K_U(t) \ge K(t) \\ \dfrac{\binom{b_{sf}}{k} \sum_{S_1 \in E : |S_1| = k} \prod_{i \in S_1} v_i(t) Sz(K(t)-k, b_{sr}, v_{T-S_1}(t))}{\sum_{k=0}^{b_{sf}} \binom{b_{sf}}{k} \sum_{S_1 \in E : |S_1| = k} \prod_{i \in S_1} v_i(t) Sz(K(t)-k, b_{sr}, v_{T-S_1}(t))} & \text{, otherwise} \end{cases}$$

Then we get: $P_{nic}(k, b_{sf}, b_{sr}, K(t), K_U(t), v(t)) =$

$$P_{cn}(k, b_{sf}, b_{sr}, K(t), K_U(t), v(t))$$
$$-P_{cn}(k+1, b_{sf}, b_{sr}, K(t), K_U(t), v(t)) .$$

- **Accounting for secure nodes**: Let s_3 be the probability that the node can receive at least one packet: $s_3 = \sum_{k=0}^{B-1} \sum_{m=0}^{B-1-k} P_j(m,k)$ and let f_c be the fraction of contaminated nodes (by packets of A):

$$\frac{df_c}{dt} = \beta(N f_c + 1) \left(1 - \frac{Ns}{N} - f_c\right) \frac{s_3}{1 - f_c} .$$

We define P_{cgi0} and P_{ucus} as the probability that a node is contaminated given that no extra-session index is in its buffer, and the probability that a node is uncontaminated and unsecured given that no extra-session index is in its buffer, respectively.

$$P_{cgi0} = \begin{cases} \frac{(f_c - \sum_{i=0}^{B} \sum_{j=1}^{B} P_j(i,j))}{\sum_{i=0}^{B} P_j(i,1)} & \text{, if } \sum_{i=0}^{B} P_j(i,1) > 0 \\ 1 & \text{, otherwise} \end{cases}$$

$$P_{ucus} = \begin{cases} \frac{1 - f_c - Ns/N}{\sum_{i=0}^{B} P_j(i,1)} & \text{, if } \sum_{i=0}^{B} P_j(i,1) > 0 \\ 0 & \text{, otherwise} \end{cases}$$

- $\mathbf{K_S(t), K_A(t), R(t), S(t)}$: Let $R(t)$ (resp. $S(t)$) be the number of indexes of S (resp. A) that D has got until time t. Let Pds (resp. Pda) be the average number of indexes of S (resp. A) that D receives per time unit around t. (The expressions of $S(t)$ is symmetric.) We have:

$$\frac{dR(t)}{dt} = \beta N P ds ,$$

$$Pds = \sum_{i=1}^{B} \sum_{j=0}^{B-i} P_j(i,j) \sum_{l=1}^{i} \sum_{n_S=l}^{i} \sum_{n_A=0}^{j} \sum_{ll=l}^{j+i} l \binom{ll}{l} \frac{\binom{n_S}{l}}{\binom{n_S+n_A}{l}} \dots$$

$$\frac{\binom{n_A}{ll-l}}{\binom{n_S+n_A-l}{ll-l}} ((n_S + n_A \ge ll) Pr[\xi = ll] + \dots$$

$$Pr[\xi > ll](n_S + n_A == ll)) \dots$$

$$P_{nic}(n_S, i, L_S(t), K_S(t), K_S(t), v(t)) \dots$$

$$P_{nic}(n_A, j, L_A(t), K_A(t), K_A(t), w(t)) .$$

Let $K_S(t)$ (resp. $K_A(t)$) be the number of indexes released by S (resp. A) up to time t, and Ps (resp. Pa) be the average number of indexes that S (resp. A) gives around time t. We have:

$$\frac{dK_S(t)}{dt} = \beta N P_S \ ,$$

$$Ps = \sum_{l=1}^{B} \sum_{i=0}^{B-l} \sum_{j=0}^{B-l-i} l \cdot P_{gs}(l, i+j) P_j(i,j) \ .$$

As well:

$$Pa = \sum_{l=1}^{B} \sum_{i=0}^{B-l} \sum_{j=0}^{B-l-i} l \cdot P_{ga}(l, i+j) P_j(j,i) \dots$$

$$((i > 0) + (i == 0) P_{cgi0} + (i == 0) P_{ucus}) \ .$$

We define $K_{US}(t)$ (resp. $K_{UA}(t)$) as the number of indexes spread by S (resp. A) up to t which have still spray-counter lower than M (resp. Q): $K_{US}(t) = \min(K_S(t), |\{I = 0, \dots, K_S(t) : \tilde{X}_I(t) < M\}|)$.All the other terms governing the dissemination of adversarial packets can be deduced from the above derivations, by accounting for possible secure nodes in the same way as done for $P_{ga}(.,.)$.

• $\mathbf{P_{gs}}$ and $\mathbf{P_{ga}}$: Let $P_{gs}(m,n)$ (resp. $P_{ga}(m,n)$) be the probability that S (resp. A) gives m indexes of S (resp. A) to a node with already n indexes of S and A. Recall that K' is the maximum number of packets S can send out, and that A sends new packets without any limit. We have:
$P_{gs}(m,n) = Pr[\min(bw, B-n, K' - K_S(t)) = m]$, $P_{ga}(m,n) = Pr[\min(bw, B-n) = m]$.

In what follows, we use the following notations. Let consider communication from node $N1$ to node $N2$. Let r be the number of indexes in $N1$ already obtained by $N2$, n_S (resp. n_A) be the number of indexes of S (resp. A) present at $N1$ but not yet at $N2$, and ll the number of indexes transferred from $N1$ to $N2$.

• $\mathbf{P_{ls}}$: Let $P_{ls}(k,j,m)$ be the probability that a (j,m)-node looses k indexes of S by meeting with D: $P_{ls}(k,j,m) = P_j(j,m) f(k,j,m)$ with

$$f(k,j,m) = \sum_{r=0}^{k} \sum_{n_S=0}^{j} \sum_{n_A=0}^{m} \sum_{ll=k-r}^{n_S+n_A} \dots$$

$$\binom{ll}{k-r} \frac{\binom{j-r}{k-r}}{\binom{j-r+n_A}{k-r}} \frac{\binom{n_A}{ll-(k-r)}}{\binom{j-r+n_A-(k-r)}{ll-(k-r)}} \dots$$

$$((j-r+n_A \geq ll) Pr[\xi = ll] + Pr[\xi > ll](j-r+n_A == ll)) \dots$$

$$P_{nic}(j-r, j, L_S(t), K_S(t), K_S(t), v(t)) \dots$$

$$P_{nic}(n_A, m, L_A(t), K_A(t), K_A(t), w(t)) \ .$$

The expression of $P_{la}(k,j,m)$ is symmetric.

• $\mathbf{P_{gsv}}$ et $\mathbf{P_{gav}}$: Let $P_{gsv}(i,j)$ be the probability that S gives at least one index to an (i,j)-node:
$P_{gsv}(i,j) = \sum_{k=1}^{B-i-j} P_{gs}(k, i+j)$, $P_{gav}(j,i) = P_j(j,i)((i > 0) + (i == 0) P_{cgi0} + (i == 0) P_{ucus}) \sum_{k=1}^{B-i-j} P_{ga}(k, j+i)$.

• $\mathbf{P_{lsv}}$: Let $P_{lsv}(i,j)$ be the probability that an (i,j)-node looses at least one index of S after meeting with D: $P_{lsv}(i,j) = P_j(i,j) \sum_{l=1}^{i} f(l,i,j)$. The expression of $P_{lav}(i,j)$ is symmetric.

• $\mathbf{P_{grs}}$: Let $P_{grs}(p,k,j)$ be the probability that a node with j index of S gains p indexes of S from a node having k index of S (resp. A). Let us define $g(p,k,m,j,l)$ as the probability that p packets exactly be transferred from a (k,m)-node to a (j,l)-node:

$$g(p,k,m,j,l) = \sum_{n_S=p}^{k} \sum_{n_A=0}^{m} \sum_{ll=p}^{n_S+n_A} \binom{ll}{p} \frac{\binom{n_S}{p}}{\binom{n_S+n_A}{p}} \frac{\binom{n_A}{ll-p}}{\binom{n_S+n_A-p}{ll-p}} \dots$$

$$((n_S + n_A \geq ll) Pr[\xi == ll] + (n_S + n_A == ll) Pr[\xi > ll]) \dots$$

$$P_{nic}(n_S, k, j, K_S(t), K_{US}(t), v(t)) \dots$$

$$P_{nic}(n_A, m, l, K_A(t), K_{UA}(t), w(t)) \ ,$$

$P_{grs}(p,k,j) = \sum_{l=0}^{B-j} \sum_{m=0}^{B-k} P_j(k,m) P_j(j,l) g(p,k,m,j,l)((m == 0)(1 - P_{cgi0}) + ((m == 0) P_{cgi0} + (m > 0))((l > 0) + (l == 0) P_{cgi0} + (l == 0) P_{ucus}))$.
The expression of $P_{gra}(p,k,j)$ can be deduced easily from that of $P_{grs}(p,k,j)$.

• $\mathbf{P_{grsv}}$: Let $P_{grsv}(k,i)$ be the probability that a node with i indexes of S gains at least one index of S by meeting with a node with k of S.
$P_{grsv}(k,i) = \sum_{j=0}^{B-i-1} \sum_{m=0}^{B-k} P_j(k,m) P_j(i,j)((m == 0)(1 - P_{cgi0}) + ((m == 0) P_{cgi0} + (m > 0))((j > 0) + (j == 0) P_{cgi0} + (j == 0) P_{ucus})) \sum_{l=1}^{k} g(l,k,m,i,j)$.

B. Per-index dissemination

$$\frac{d\tilde{X}_I}{dt} = \beta N N A_R(I) + \beta N c_2 A_S - \beta \tilde{X}_I A_D$$

Let us first define $ptnh(I, i, K(t))$ as being the probability for a node with i indexes of S to not have index I of S in its buffer (notations are the same as those for the definition of P_{nic}):

$$ptnh(I, i) = Sz(K(t) - 1, i, v_{T-\{I\}}(t)) / Sz(K(t), i, v(t)) \ .$$

$A_R(I)$ is equal to the fraction of nodes without index I, that get it from another relay. The expression of $A_R(I)$ below accounts for the spray-and-wait mechanism which sorts out in ascending order the spray counters before choosing the first corresponding indexes to forward. Here, we approximate the local counters available at each node by the global counters $\tilde{X}_I(t)$ from the network analysis. Let us define $h(k,m,i,j)$ as the probability that index I of S gets forwarded from a (k,m)-node to a (i,j)-node:

$$h(k,m,i,j) = \sum_{l=1}^{k} \sum_{n_S=l}^{k} \sum_{n_A=0}^{m} \sum_{ll=l}^{n_S+n_A} \frac{P_c(I)}{sumhnh(k,i)} \binom{ll}{l} \dots$$

$$\frac{\binom{n_S}{l}}{\binom{n_S+n_A}{l}} \frac{\binom{n_A}{ll-l}}{\binom{n_S+n_A-l}{ll-l}} ((n_S + n_A \geq ll) Pr[\xi = ll] \dots$$

$$+ Pr[\xi > ll](n_S + n_A == ll)) P_{nic}(n_S, k, i, K_S(t), K_{US}(t), v(t)) \dots$$

$$P_{nic}(n_A, m, j, K_A(t), K_{UA}(t), w(t))((m == 0) \dots$$

$$(1 - P_{cgi0}) + ((m == 0) P_{cgi0} + (m > 0))((j > 0) \dots$$

$$+ (j == 0) P_{cgi0} + (j == 0) P_{ucus})) \ ,$$

$A_R(I) =$

$$\begin{cases} 0 \quad , \text{if } \tilde{X}_I \geq M \\ \sum_{k=1}^{B} \sum_{m=0}^{B-k} P_j(k,m)(1 - pthnh(I,k)) \ldots \\ \sum_{i=0}^{B-1} \sum_{j=0}^{B-1-i} P_j(i,j) ptnh(I,i) h(k,m,i,j), \text{ otherwise} \end{cases}$$

with the definitions of $sumhnh(I,k,i)$ and $P_c(I)$ as follows: $sumhnh(k,i) = \sum_I ptnh(I,i)(1 - ptnh(I,k)$. $P_c(I)$ is the probability that I gets forwarded (replicated) given that I pertains to the set of n_S indexes not in common with the receiving node, given that $\tilde{X}_I < M$ and given that l packets gets forwarded. Let $C_{(l)}$ be the l-th order statistic of a set of $n = n_S - 1$ elements following a discrete distribution $Pr[X = x]$ such that: for $a = 0, \ldots, M-1$, $Pr[X = a] = \frac{|\{I: round(\tilde{X}_I) = a\}|}{|\{I: round(\tilde{X}_I) < M\}|}$. Then the cdf of $C_{(l)}$ is $Pr[C_{(l)} < x] = \sum_{j=0}^{n-l} \binom{n}{j} (Pr[X \geq x])^j (Pr[X < x])^{n-j}$. Accounting for the case when several indexes have the same number of copies which turn to be the l-th minimum, we get:

$$P_c(I) = Pr[C_{(l)} \geq \tilde{X}_I(t)] \min(1, \ldots$$
$$\frac{l - (n_S - 1) Pr(X < \tilde{X}_I)}{1 + (n_S - 1) Pr[C_{(l)} = \tilde{X}_I(t)] / Pr[C_{(l)} \geq \tilde{X}_I(t)]}) \ .$$

A_S is the activation of index I: I is sent only once, and indexes are sent from $I = 1$ to $I = K'$.
$A_S = (K_S(t) \leq I)(K_S(t) + B > I)(K_S(t) \leq K')$,
$c_2 = \sum_{i=0}^{B-1} \sum_{j=0}^{B-1-i} P_j(i,j) ptnh(I,i) Pr[\xi > I - K_S(t)](B - (i+j) > I - K_S(t))$.

A_D is the fraction of nodes holding I that loose it (by meeting with D).

$$A_D = \sum_{i=1}^{B} \sum_{j=0}^{B-i} P_j(i,j) \sum_{l=1}^{i} \sum_{r=0}^{l-1} \sum_{n_A=0}^{j} \sum_{n_S=i-r}^{i-r} \sum_{ll=l-r}^{n_S+n_A} \ldots$$
$$[\frac{l}{sumhnh(i, L_S(t)}(1 - P_I(t))(1 - ptnh(I,i)) \ldots$$
$$+ \frac{r}{sumhh(i, L_S(t)} P_I(t)(1 - ptnh(I,i))] \ldots$$
$$\binom{ll}{l-r} \frac{\binom{n_S}{l-r}}{\binom{n_S+n_A}{l-r}} \frac{\binom{n_A}{ll-(l-r)}}{\binom{n_S+n_A-(l-r)}{ll-(l-r)}} ((n_S + n_A \geq ll) Pr[\xi = ll] \ldots$$
$$+ Pr[\xi > ll](n_S + n_A == ll)) P_{nic}(n_S, i, L_S(t), K_S(t), K_S(t), v(t))$$
$$\ldots P_{nic}(n_A, j, L_A(t), K_A(t), K_A(t), w(t)) \ ,$$

with the following definitions: $sumhnh(i, L_S(t)) = \sum_I (1 - P_I(t))(1 - ptnh(I,i))$, $sumhh(i, L_S(t)) = \sum_I P_I(t)(1 - ptnh(I,i))$ and $P_I(t)$ below.

Let $P_I(t)$ be the probability that the destination has received index I of S by time t:

$$\frac{dP_I(t)}{dt} = \beta A'_D (1 - P_I(t))$$

where A'_D is the fraction of nodes holding I that give it to D, provided that D does not have I yet.

$$A'_D = \sum_{i=1}^{B} \sum_{j=0}^{B-i} P_j(i,j)(1 - ptnh(I,i)) \sum_{l=1}^{i} \sum_{n_S=l}^{i} \sum_{n_A=0}^{j} \sum_{ll=l}^{n_S+n_A} \ldots$$
$$\frac{P_c(I)}{sumhnh(i, L_S(t))} n_S \binom{ll}{l} \frac{\binom{n_S}{l}}{\binom{n_S+n_A}{l}} \frac{\binom{n_A}{ll-l}}{\binom{n_S+n_A-l}{ll-l}} \ldots$$
$$((n_S + n_A \geq ll) Pr[\xi = ll] + Pr[\xi > ll](n_S + n_A == ll)) \ldots$$
$$P_{nic}(n_S, i, ltS, K_S(t), K_S(t), v(t)) \ldots$$
$$P_{nic}(n_A, j, L_A(t), K_A(t), K_A(t), w(t)) \ .$$

For other sessions, $Q_I(t)$ is the probability that the destination has received index I of A by time t. The expression is symmetric to $P_I(t)$.

Let $K_{Amax}(T)$ be the maximum number of indexes A can spread out during time T. Once $P_I(t)$ are obtained for all $t = 0, \ldots, T$ and $I = 1, \ldots, K'$ and $Q_I(t)$ for $I = 1, \ldots, K_{Amax}(T)$, the distributions of the dimensions of the vector spaces of S and A retrieved at D are given by the distributions of the number $\rho(t)$ and $\sigma(t)$ of indexes received from S and A, respectively. As RLCs at sources are assumed to lie in a high-order finite field, the number of DoF received at S is the minimum between the number of received indexes and K. The means of $\rho(t)$ and $\sigma(t)$ are $R(t)$ and $S(t)$ defined above, and we have, for all $z = 0, \ldots, K'$ and $x = 1, \ldots, K_{Amax}(T)$:

$$Pr[\rho(t) = z] = S_Z(K', z, \{P_I(t)\}_{I=1 \ldots K'})$$
$$Pr[\sigma(t) = x] = S_Z\left((K_{Amax}(T), x, \{Q_I(t)\}_{I=1 \ldots K_{Amax}(T)}\right) \ .$$

V. Numerical results

The accuracy of the above analysis based on fluid models is assessed in this section. The simulations are performed with Matlab, on a synthetic contact trace whose inter-meeting intensity is $\beta = 5.10^{-5}$ and the number of nodes is $N = 1000$. We make vary the number of nodes not allowed to relay other sessions such that $N_s = 1000$ and $N_s = 500$. Each point is obtained by averaging over 300 runs.

We first consider no restriction on the number of copies that can be spread, namely $M = N$, with $N_s = 1000$. Fig. 1 compares the evolution of the analytical prediction of the mean numbers of indexes received from S ($R(t)$) by D against the experiments. In Fig. 2, the distribution of this quantity, i.e., of $\rho(t)$ is compared against the empirical density for one time-shot.

For number of copies limited to $M = Q = 5$ or $M = Q = 10$, Fig. 3 and 4 for $N_s = 1000$, and Fig. 5 and 6 for $N_s = 500$ represent the same quantities as above. We hereby verify the good fit between analytical prediction and experiment, both for mean values and for complete distributions.

VI. Conclusion

In this paper, we have carried out the performance analysis of Spray-and-Wait routing in DTN, in the presence of background traffic that can be network-coded with the session of interest. We have used fluid models and have been able to derive the distribution of the number of decodable packets received at the destination. We have validated the derived model on synthetic traces. Note that the complexity of the model depends only on the buffer size, and the number of

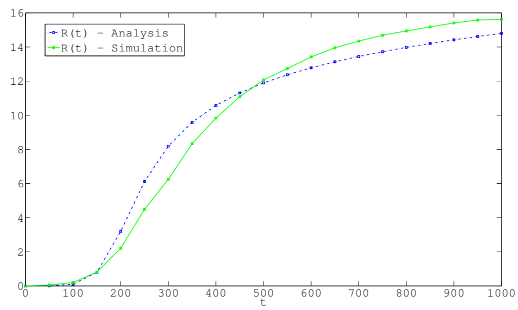

Fig. 1. Mean number $R(t)$ of indexes of the session of interest received by D by time t, $Ns = 1000$, $M = Q = N$.

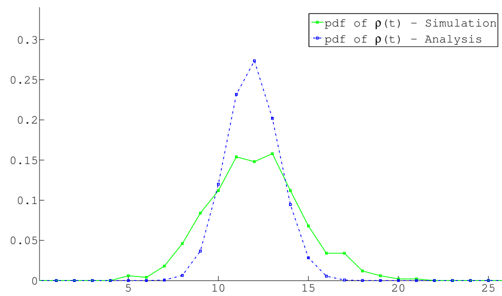

Fig. 2. Distributions of the numbers $\rho(t)$of indexes of the session of interest received by D by time $t = 500$, $Ns = 1000$, $M = Q = N$.

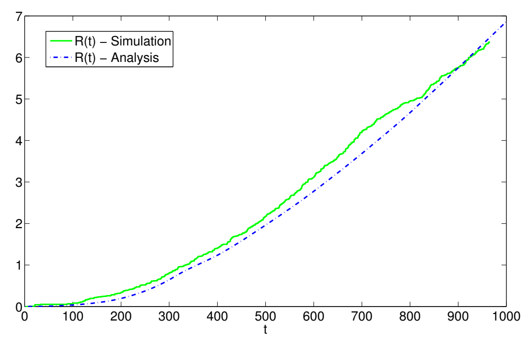

Fig. 3. Mean number $R(t)$ of indexes of the session of interest received by D by time t, $Ns = 1000$, $M = Q = N$.

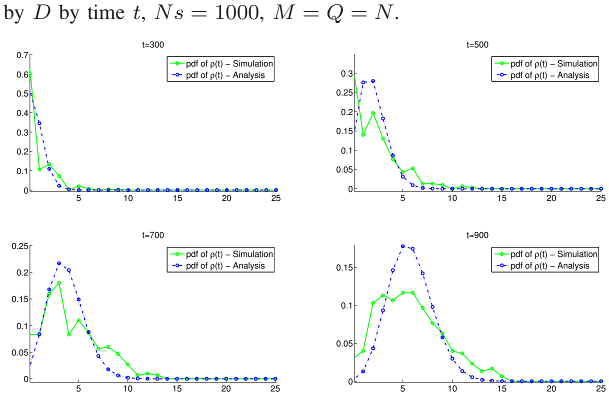

Fig. 4. Distributions of the numbers $\rho(t)$ of indexes of the session of interest received by D by time t, $Ns = 1000$, $M = 5$.

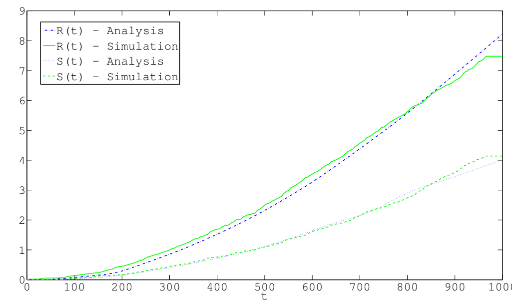

Fig. 5. Mean numbers $R(t)$ and $S(t)$ of indexes of the session of interest and others received by D by time t, $Ns = 500$, $M = Q = 10$.

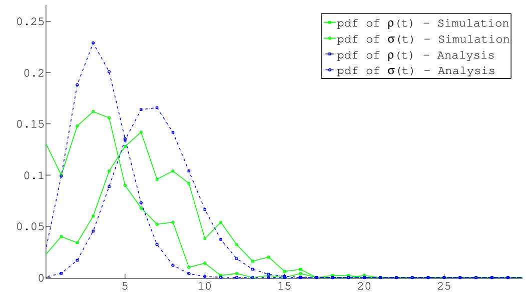

Fig. 6. Distributions of the numbers $\rho(t)$ and $\sigma(t)$ of indexes of the session of interest and others received by D by time $t = 900$, $Ns = 500$, $M = Q = 10$.

packets the sources can send. However, heuristics to decrease the computational intensity of the model will be presented along the other following extensions.

Future work will address the extension of this model to the case of heterogeneous mobility, so as get performance prediction of the efficient Spray-and-Wait routing with a more realistic network model. This will allow to optimize the share of the total energy (spreading) budget amongst the nodes according to their mobility features.

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