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1 Formalization

Given the following symbols and sentences:

- f = "John is fit"
- l = "John is lucky"
- w = "the weather is good"
- c = "John succeeds in climbing mount Everest"

Formalize the following sentences in propositional logic:

- 1. John is fit and lucky
- 2. John is fit but not lucky, and the weather is not good.
- 3. If John is not fit and lucky, then he does not succeed in climbing mount Everest.
- 4. John succeeds in climbing mount Everest if he is fit and if the weather is good
- 5. If the weather is not good but John is lucky and fit, he succeeds in climbing mount Everest
- 6. If it is the case that the weather is good provided that John is lucky, then John succeeds in climbing mount Everest if he is fit and lucky.

2 Evaluation of Propositional Sentences

Let \mathcal{I} be defined as follows:

$$\begin{array}{c} \text{constant} & p & q & r \\ \text{truth value} & F & F & T \end{array}$$

Evaluate the following sentences:

1.
$$p \Rightarrow q$$

- 2. $q \Rightarrow r$
- 3. $r \Rightarrow p$
- 4. $\neg(q \Leftrightarrow r) \lor p$
- 5. $(p \Rightarrow (q \Rightarrow r)) \Leftrightarrow ((p \land q) \Rightarrow r)$

6.
$$(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$$

7. $\neg q \land (p \lor q) \land (q \lor r) \land (p \Rightarrow \neg r)$

3 Truth Tables

For each of the following sentences, list their properties (i.e., are they valid, satisfiable, contingent, falsifiable, or unsatisfiable):

1. $p \Rightarrow q$ 2. $q \Rightarrow r$ 3. $r \Rightarrow p$ 4. $\neg (q \Leftrightarrow r) \lor p$ 5. $(p \Rightarrow (q \Rightarrow r)) \Leftrightarrow ((p \land q) \Rightarrow r)$ 6. $(p \Rightarrow q) \Leftrightarrow (\neg q \Rightarrow \neg p)$ 7. $\neg q \land (p \lor q) \land (q \lor r) \land (p \Rightarrow \neg r)$

4 Entailment

Check whether

$$\{\neg p \Rightarrow q, q \Rightarrow p, p \Rightarrow (r \land s)\} \models p \land r \land s$$

- 1. With the truth table method.
- 2. By refutation, with the Davis-Putnam Algorithm
- 3. By refutation, with clausal resolution

Compare the three approaches.

Suggestion: you will need a CNF to prove the entailment by refutation; just construct a single sentence from the three premises, then transform that sentence into CNF.