Logic for AI — Master 1 IFI
Class Assignment #2: Predicate Logic

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1 Formalization

Translate the following English sentences into predicate logic sentences:

1. Every person who owns an apartment pays the real estate tax.

2. A hotel is rated one-star if all rooms have shower and WC.

3. A person is a great parent if they have a child who has children.

2 Nomenclature

Tell which qualifications among *an atom*, *a literal*, *ground*, *open*, and *closed* apply to the following sentences:

1. $P(x) \lor Q(a)$,

2. $R(x, y)$,

3. $\neg\neg R(a, b)$,

4. $\forall x \exists y R(x, y)$,

5. $P(a) \land R(a, b)$,

6. $\neg P(a)$,

7. $\neg P(x)$,

8. $P(a) \land \exists x R(a, x)$.

3 Herbrand Base

Write the Herbrand base for the following languages:

1. $\{a, P(\cdot), Q(\cdot)\}$,

2. $\{0, 1, \mathsf{Even}(\cdot), \mathsf{Lt}(\cdot, \cdot)\}$,

3. $\{a, b, f(\cdot), g(\cdot, \cdot), P(\cdot, \cdot, \cdot)\}$.

4   Herbrand Models

Given the Herbrand model $\mathcal{I} = \{P(a), Q(b), R(a, b), S(a, a), S(b, b)\}$, evaluate the following sentences:

1. $\exists x R(a, x)$,

2. $\forall x \exists y R(x, y)$,

3. $\forall x P(x) \Rightarrow Q(x)$,

4. $\forall x S(x, x)$.

5   Herbrand Entailment

Which of the following entailments hold, using Herbrand semantics?

1. $Q(a) \models \forall x (P(x) \Rightarrow Q(x))$,

2. $P(a) \models \neg Q(b) \wedge \exists x (P(x) \wedge Q(x))$.

6   Semantic Trees

Check the Herbrand satisfiability of the following sentence by applying Herbrand's Theorem:

$$\neg \left(\forall x P(x, a, g(x, b)) \Rightarrow \exists y \exists x P(f(y), z, g(f(a), b)) \right).$$