Logic for AI — Master 1 IFI Class Assignment #6: Possibility Theory

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1 Possibility Measures

Let the universe Ω contain the following four interpretations:

$$\begin{array}{rcl} \omega_0 &=& \{P\mapsto 0, Q\mapsto 0\},\\ \omega_1 &=& \{P\mapsto 0, Q\mapsto 1\},\\ \omega_2 &=& \{P\mapsto 1, Q\mapsto 0\},\\ \omega_3 &=& \{P\mapsto 1, Q\mapsto 1\}. \end{array}$$

Let us assume the possibility distribution π is given, such that

$$\pi(\omega_0) = 0.3,$$

 $\pi(\omega_1) = 1,$
 $\pi(\omega_2) = 0.1,$
 $\pi(\omega_3) = 0.2.$

Compute the following:

- 1. $\Pi(P)$;
- 2. N(P);
- 3. $\Pi(P \Rightarrow Q)$;
- 4. $N(P \Rightarrow Q)$.

2 Semantics of a Possibilistic Belief Base

Let $\Sigma = \{(P \Rightarrow Q, 0.6), (P \lor R, 1), (\neg R, 0.2)\}$ be a possibilistic belief base. Compute its associated possibility distribution π_{Σ} .

3 Guaranteed Possibility

Given a possibility distribution π , a guaranteed possibility measure, noted Δ , is defined as:

$$\Delta(\phi) = \min_{\omega \models \phi} \pi(\omega). \tag{1}$$

In words, the guaranted possibility measure estimates to what extent *all* the models of ϕ are actually possible according to what is known, i.e., any model of ϕ is at least possible to degree $\Delta(\phi)$.

Derive the properties of Δ .