

Logic for AI — Master 1 IFI

Class Assignment #2: Predicate Logic

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1 Formalization

Translate the following English sentences into predicate logic sentences:

1. Every person who owns an apartment pays the real estate tax.
2. A hotel is rated one-star if all rooms have shower and WC.
3. A person is a great parent if they have a child who has children.

2 Nomenclature

Tell which qualifications among *an atom*, *a literal*, *ground*, *open*, and *closed* apply to the following sentences:

1. $P(x) \vee Q(a)$,
2. $R(x, y)$,
3. $\neg\neg R(a, b)$,
4. $\forall x \exists y R(x, y)$,
5. $P(a) \wedge R(a, b)$,
6. $\neg P(a)$,
7. $\neg P(x)$,
8. $P(a) \wedge \exists x R(a, x)$.

3 Herbrand Base

Write the Herbrand base for the following languages:

1. $\{a, P(\cdot), Q(\cdot)\}$,
2. $\{0, 1, \text{Even}(\cdot), \text{Lt}(\cdot, \cdot)\}$,
3. $\{a, b, f(\cdot), g(\cdot, \cdot), P(\cdot, \cdot, \cdot)\}$.

4 Herbrand Models

Given the Herbrand model $\mathcal{I} = \{P(a), Q(b), R(a, b), S(a, a), S(b, b)\}$, evaluate the following sentences:

1. $\exists x R(a, x)$,
2. $\forall x \exists y R(x, y)$,
3. $\forall x P(x) \Rightarrow Q(x)$,
4. $\forall x S(x, x)$.

5 Herbrand Entailment

Which of the following entailments hold, using Herbrand semantics?

1. $Q(a) \models \forall x (P(x) \Rightarrow Q(x))$,
2. $P(a) \models \neg Q(b) \wedge \exists x (P(x) \wedge Q(x))$.

6 Semantic Trees

Check the Herbrand satisfiability of the following sentence by applying Herbrand's Theorem:

$$\neg (\forall x P(x, a, g(x, b)) \Rightarrow \exists y \exists z P(f(y), z, g(f(a), b))).$$