Logic for AI — Master 1 IFI
Class Assignment #5: Fuzzy Logic

Andrea G. B. Tettamanzi
Université côte d'Azur
andrea.tettamanzi@univ-cotedazur.fr

Academic Year 2019/2020

1 Fuzzy Sets

Using your intuition and triangular, trapezoidal, or gaussian functions, define the membership functions on the real line for the following fuzzy sets:

1. the fuzzy number "approximately 3";
2. the fuzzy number "approximately 2 or approximately 8";
3. the fuzzy number "approximately 6 to approximately 8";
4. for the weight of people: very light, light, average, heavy, very heavy;
5. for outdoor temperature: cold, cool, comfortable, warm, hot;
6. for alcohol content of a beverage: soft, light, hard.

2 Operations on Fuzzy Sets

Let

$$\begin{aligned} A &= \frac{1}{a} + \frac{0.7}{b} + \frac{0.4}{c} + \frac{0.2}{d} \\ B &= \frac{0.5}{b} + \frac{1}{c} + \frac{0.5}{d} + \frac{0.1}{e} \end{aligned}$$

be fuzzy sets defined on the universe $U = \{a, b, c, d, e\}$. Compute (using the min t-norm and max t-conorm):

1. $A \cup B$;
2. $A \cap B$;
3. $\overline{A}$.
4. $\overline{B}$.
5. $A \cap \overline{B}$.
6. $A \cap \overline{A}$.

3 Extension Principle

Let $f : U \times U \to \{0, 1, 2, 3, 4\}$, where U is defined like in Exercise 2 and let $f(x, y)$ be given by the following table:

$x \setminus y$	a	b	c	d	e
a	0	1	2	3	4
b	1	1	2	3	4
c	2	2	2	3	4
d	3	3	3	3	4
e	4	4	4	4	4

Use the Extension Principle to compute the fuzzy set $f(A, B)$, where A and B are the sets defined in Exercise 2.

4 Fuzzy Inference

Consider the following rule base:

IF	x is left	AND	v is neg	THEN	F is pos
IF	x is left	AND	v is pos	THEN	F is zero
IF	x is right	AND	v is neg	THEN	F is zero
IF	x is right	AND	v is pos	THEN	F is neg

and let the linguistic values be defined as follows:

- for variable x:

$$\text{left}(x) = \begin{cases} 1, & x \leq -1; \\ (1-x)/2, & -1 < x < 1; \\ 0, & x \geq 1; \end{cases} \qquad \text{right}(x) = \begin{cases} 0, & x \leq -1; \\ (x+1)/2, & -1 < x < 1; \\ 1, & x \geq 1; \end{cases}$$

- for variable v:

$$\text{neg}(v) = \begin{cases} 1, & v \leq -1; \\ (1-v)/2, & -1 < v < 1; \\ 0, & v \geq 1; \end{cases} \qquad \text{pos}(v) = \begin{cases} 0, & v \leq -1; \\ (v+1)/2, & -1 < v < 1; \\ 1, & v \geq 1; \end{cases}$$

- for variable F:

$$\text{zero}(F) = \begin{cases} 1 - |F|, & |F| < 1; \\ 0, & |F| \geq 1; \end{cases}$$

$$\text{neg}(F) = \begin{cases} 1, & F \leq -1; \\ -F, & -1 < F < 0; \\ 0, & F \geq 0; \end{cases} \qquad \text{pos}(F) = \begin{cases} 0, & F \leq 0; \\ F, & 0 < F < 1; \\ 1, & F \geq 1. \end{cases}$$

Compute the fuzzy set on F when $x = -0.5$ and $v = -0.1$.