Master in Computer Science, 1st Year
Logic for AI

Written Final Test

Tuesday, January 8, 2019

Please write clearly. Answers to exercises should be justified. Clearly mark your copy with your name. Only a single handwritten A4 sheet of notes is allowed. No books, documents other than the handwritten A4 sheet, computers, tablets, cellphones, or smartphones are allowed. The grading scheme, on a 0 to 20 scale, is provided for information only.

1. (10 points) Given the premises

   (a) $\forall x(INVESTOR(x) \Rightarrow \exists y((STOCK(y) \vee BOND(y)) \wedge BUY(x,y)))$
   (Every investor bought [something that is] stocks or bonds).

   (b) $DJCRASH \Rightarrow \forall x((STOCK(x) \wedge \neg GOLD(x)) \Rightarrow FALL(x))$
   (If the Dow-Jones Average crashes, then all stocks that are not gold stocks fall).

   (c) $TBRISE \Rightarrow \forall x(BOND(x) \Rightarrow FALL(x))$
   (If the T-Bill interest rate rises, then all bonds fall).

   (d) $\forall x \forall y(INVESTOR(x) \wedge BUY(x,y) \wedge FALL(y) \Rightarrow \neg HAPPY(x))$
   (Every investor who bought something that falls is not happy).

   Prove, by resolution,

   $(DJCRASH \wedge TBRISE) \Rightarrow \forall x(INVESTOR(x) \wedge HAPPY(x) \Rightarrow \exists y(GOLD(y) \wedge BUY(x,y)))$.

2. (10 points) Let the mapping $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ be defined as $f(x,y) = x \pmod{y}$.

   Use the Extension Principle to compute the fuzzy set $f(A,B)$, where A and B are fuzzy sets on $\mathbb{N}$ defined as follows:

$$\begin{aligned} A &= \frac{0.4}{21} + \frac{0.7}{22} + \frac{1}{23} + \frac{0.6}{24} + \frac{0.2}{25}, \\ B &= \frac{0.3}{2} + \frac{1}{3} + \frac{0.5}{4}. \end{aligned}$$