Logic for AI

Master 1 Informatique

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Unit 1

Propositional Logic

Agenda

- Introduction
- Propositional Logic
  - Syntax
  - Semantics

Introduction

- One of the hallmarks of **intelligence** is the ability to reason
- If we want to build **intelligent machines**, we must be able to automate **reasoning**
- Logic is the **study of how we** (should) **reason**
- One of the oldest intellectual disciplines in human history
  - Aristotle (Ἀριστοτέλης, 384–322 BC), a pupil of Plato
  - Gottfried Wilhelm von Leibniz (1646–1716)
  - George Boole (1815–1864)
  - Bertrand Russel (1872–1970)
  - Alan Turing (1912–1954)
  - … and many others!

Introduction

- Logic plays an important role in several areas of CS
  - software engineering (specification and verification)
  - programming languages (semantics, logic programming)
  - **artificial intelligence** (knowledge representation and reasoning).
- Goals of this course
  - Provide general background in Logic
  - Enable access to more advanced topics in CS
  - In particular, (symbolic) artificial intelligence
  - Deal with uncertainty, imprecision, and incompleteness

Contents of the Course

- Part I – Basics
  - Propositional Logic: syntax and semantics
  - First Order Predicate Logic: syntax and semantics
  - Natural Deduction
  - Unification and Resolution

- Part II – Non-Monotonic Logic and Approximate Reasoning
  - Fuzzy Logic
  - Possibility Theory
  - Belief Revision and Update
  - Argumentation Theory

Credits

I’m indebted to many colleagues. In particular:

- Michael Genesereth & Eric Kao (Stanford)
- Patrice Clemente (ENSI Bourges)

What is Logic?

- Logic is the study of information encoded in the form of logical sentences (or formulas).
- Each sentence *S* divides the set of possible worlds into
  - The set of worlds in which *S* is true (models of *S*)
  - The set of worlds in which *S* is false (counter-models of *S*)
- A set of premises logically entails a conclusion ⇔ the conclusion is true in every world in which all of the premises are true
- A logic consists of
  - A language with a formal syntax and a precise semantics
  - A notion of logical entailment
  - Rules for manipulating expressions in the language.

Why Do We Need “Formal” Logic?

- Why not study Logic using just natural language?
  - Natural language can be ambiguous
    - The boy saw the girl with the telescope
    - British Left Waffles on Falkland Islands
  - Long sentences may be too complex
  - Failing to understand the meaning of a sentence can lead to errors in reasoning
    - Bad sex is better than nothing.
      Nothing is better than good sex.
      Therefore, bad sex is better than good sex”
- These difficulties can be eliminated by using a formal language

Propositional Languages

- A propositional signature is a set of primitive symbols, called propositional constants.
- A propositional constant symbolizes a simple sentence, like
  - “it is raining” → *r*
  - “the tank is empty” → *e*
- Given a propositional signature, a propositional sentence is either
  - a member of the signature or
  - a compound expression formed from members of the signature. (Details to follow.)
- A propositional language is the set of all propositional sentences that can be formed from a propositional signature.

Compound Sentences

- Negations: ¬*raining*

  The argument of a negation is called the *target*.
- Conjunctions: (*raining* ∧ *snowing*)

  The arguments of a conjunction are called *conjuncts*.
- Disjunctions: (*raining* ∨ *snowing*)

  The arguments of a disjunction are called *disjuncts*.
- Implications: (*raining* ⇒ *cloudy*)

  The left argument of an implication is the *antecedent*.

  The right argument is the *consequent*.
- Equivalences: (*cloudy* ⇔ *raining*)

Propositional Interpretation

- A propositional interpretation is a function mapping every propositional constant in a propositional language to the truth values T or F.

$$\mathcal{I} : \text{Constants} \to \{F, T\}$$

$$\begin{array}{ccc} p & \stackrel{\mathcal{I}}{\mapsto} & T \\ q & \stackrel{\mathcal{I}}{\mapsto} & F \\ r & \stackrel{\mathcal{I}}{\mapsto} & T \end{array} \qquad \begin{array}{ccc} p^{\mathcal{I}} & = & T \\ q^{\mathcal{I}} & = & F \\ r^{\mathcal{I}} & = & T \end{array}$$

- We sometimes view an interpretation as a Boolean vector of values for the items in the signature of the language (when the signature is ordered): *TFT*

Sentential Interpretation

- A sentential interpretation is a function mapping every propositional sentence to the truth values T or F.

$$\begin{aligned} p^{\mathcal{I}} &= T & (p \vee q)^{\mathcal{I}} &= T \\ q^{\mathcal{I}} &= F & (\neg q \vee r)^{\mathcal{I}} &= T \\ r^{\mathcal{I}} &= T & ((p \vee q) \wedge (\neg p \vee r))^{\mathcal{I}} &= T \end{aligned}$$

- A propositional interpretation defines a sentential interpretation by application of operator semantics.

Operator Semantics

ϕ	$\neg\phi$
F	T
T	F

ϕ	ψ	$\phi \wedge \psi$
F	F	F
F	T	F
T	F	F
T	T	T

ϕ	ψ	$\phi \vee \psi$
F	F	F
F	T	T
T	F	T
T	T	T

ϕ	ψ	$\phi \Rightarrow \psi$
F	F	T
F	T	T
T	F	F
T	T	T

ϕ	ψ	$\phi \Leftrightarrow \psi$
F	F	T
F	T	F
T	F	F
T	T	T

Multiple Interpretations

- Logic does not prescribe which interpretation is "correct". In the absence of additional information, one interpretation is as good as another.
- Examples:
  - Different days of the week
  - Different locations
  - Beliefs of different people
- We may think of each interpretation as a *possible world*
- The set of all interpretations (possible worlds) is

$$\Omega = \{F, T\}^{\text{Constants}} \qquad \|\Omega\| = 2^{\|\text{Constants}\|}$$

Truth Tables

- A truth table is a table of all possible interpretations for the propositional constants in a language (i.e., a representation of Ω).

p	q	r
F	F	F
F	F	T
F	T	F
F	T	T
T	F	F
T	F	T
T	T	F
T	T	T

One row per interpretation

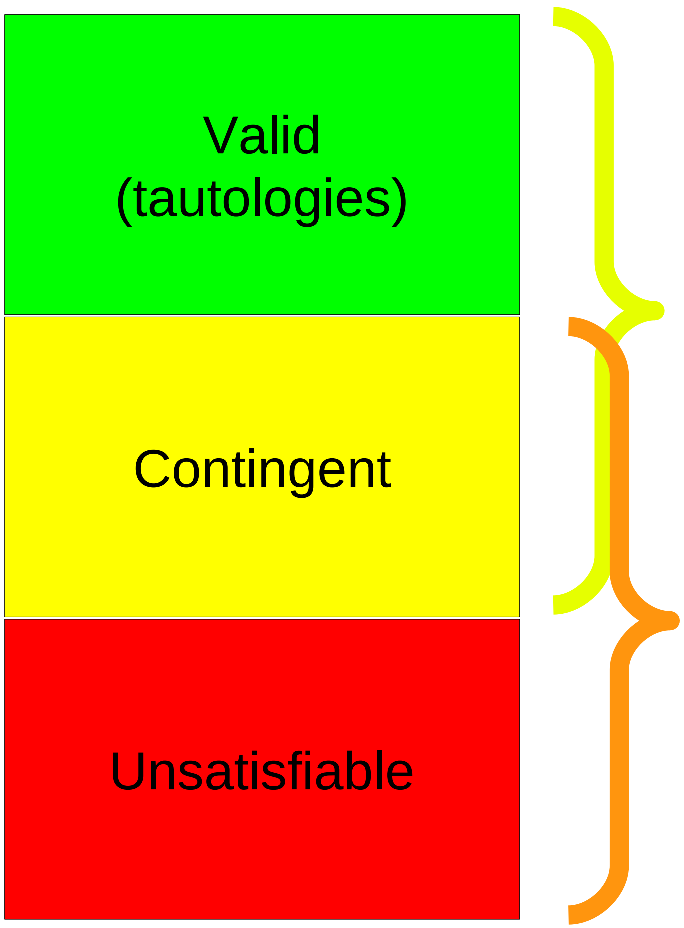
One column per constant

For a language with *n* constants, there are 2^n interpretations

Properties of Sentences

Valid (tautologies)	A sentence is *valid* if and only if *every* interpretation satisfies it.
Contingent	A sentence is *contingent* if and only if *some* interpretation satisfies it and *some* interpretation falsifies it.
Unsatisfiable	A sentence is *unsatisfiable* if and only if *no* interpretation satisfies it.

Properties of Sentences



A sentence is *satisfiable* if and only if it is either valid or contingent.

A sentence is *falsifiable* if and only if it is either contingent or unsatisfiable.

Example of a Tautology

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \vee (q \Rightarrow r)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example of a Tautology

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \vee (q \Rightarrow r)$
F	F	F	T	T	
F	F	T	T	T	
F	T	F	T	F	
F	T	T	T	T	
T	F	F	F	T	
T	F	T	F	T	
T	T	F	T	F	
T	T	T	T	T	

Example of a Tautology

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \vee (q \Rightarrow r)$
F	F	F	T	T	T
F	F	T	T	T	T
F	T	F	T	F	T
F	T	T	T	T	T
T	F	F	F	T	T
T	F	T	F	T	T
T	T	F	T	F	T
T	T	T	T	T	T

More Valid Sentences (Tautologies)

Double Negation: $p \Leftrightarrow \neg\neg p$

deMorgan's Laws:

$$\neg(p \wedge q) \quad \Leftrightarrow \quad \neg p \vee \neg q$$
$$\neg(p \vee q) \quad \Leftrightarrow \quad \neg p \wedge \neg q$$

Implication Introduction: $p \Rightarrow (q \Rightarrow p)$

Implication Distribution:

$$(p \Rightarrow (q \Rightarrow r)) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$$

Axiomatizability

- A set of boolean vectors of length *n* is *axiomatizable* in propositional logic if and only if there is a signature of size *n* and a set of sentences from the corresponding language such that the vectors in the set correspond to the set of interpretations satisfying the sentences.
- A set of sentences defining a set of vectors is called the *axiomatization* of the set of vectors.
- Example:
  - Set of Boolean Vectors: { TFF, FTF, FTT }
  - Signature: $\{p, q, r\}$
  - Axiomatization: $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q)$

Thank you for your attention