Logic for AI

Master 1 Informatique

Andrea G. B. Tettamanzi
Laboratoire I3S – Pôle SPARKS
andrea.tettamanzi@univ-cotedazur.fr

Unit 11

Belief Revision

Agenda

- Introduction
- Preliminaries
- Rationality Postulates
- Models and Representation
- Epistemic Entrenchment

Motivating Example

- Suppose we have a knowledge base containing:
  - A: Gold can only be stained by aqua regia
  - B: The acid in the bottle is sulfuric acid
  - C: Sulfuric acid is not aqua regia
  - D: My wedding ring is made of gold
- The following fact is derivable from A–D:
  - E: My wedding ring will not be stained by the acid in the bottle
- Now, suppose that, *as a matter of fact,* the wedding ring is indeed stained by the acid: you want to add ¬E to the KB
- However, the KB would become inconsistent: you have to revise
- Instead of giving up all your beliefs, you have to choose

Methodological Questions

- How are the beliefs in the knowledge base represented?
- What is the relation between the elements explicitly represented in the database and the beliefs that may be *derived* from these elements?
- How are the choices concerning how to retract made?

When beliefs are represented by sentences in a belief system K, one can distinguish three main kinds of belief changes:

- Expansion: a new sentence A together with its logical consequences is added to K: K’ = K + A
- Revision: a new sentence A is added but others must be retracted to maintain consistency: K’ = K*A
- Contraction: a sentence is retracted: K’ = K – A

Expansion

- Expansion of beliefs can be handled comparatively easily
- K + A can simply be defined as the logical closure of K with A:

$$K + A = \{B : K \cup \{A\} \models B\}$$

Introduction

- It is not possible to give a similar explicit definition of revision and contraction
- When tackling the problem of Belief Revision (and contraction), there are two general strategies to follow:
  - To present explicit **constructions** of the revision process
  - To formulate **postulates** for such constructions
- Constructions and postulates can be connected via a number of **representation theorems**

- [Peter Gärdenfors. Belief Revision: A vade-mecum, META 1992]

Preliminaries

- To simplify things, we may work in propositional logic
- The simplest way of modeling a belief state is to represent it as a set of sentences
- We define a **belief set** as a set K of sentences such that

if $K \models B$ then $B \in K$

$$Cn(K) = \{A : K \models A\}$$

There is exactly one inconsistent belief set, namely the set of all sentences in the language

Rationality Postulates (AGM)

- AGM = Alchourrón, Gärdenfors, and Makinson
- Let us assume belief sets are used as models of belief states
- AGM Postulates for rational functions of
  - Revision (*)
  - Contraction (–)
- The postulates state conditions that any rational function should satisfy
  - For all belief sets K
  - For all sentences A and B

AGM Basic Postulates for Revision

(K*1) $K * A$ is a belief set

(K*2) $A \in K * A$

(K*3) $K * A \subseteq K + A$

(K*4) If $\neg A \notin K$ then $K + A \subseteq K * A$

(K*5) $K * A = K_\perp$ if and only if $\models \neg A$

(K*6) If $\models A \Leftrightarrow B$ then $K * A = K * B$

AGM Postulates for Composite Revision

(K*7) $K * (A \wedge B) \subseteq (K * A) + B$

(K*8) If $\neg B \notin K * A$ then $(K * A) + B \subseteq K * (A \wedge B)$

AGM Basic Postulates for Contraction

(K–1) $K - A$ is a belief set

(K–2) $K - A \subseteq K$

(K–3) If $A \notin K$ then $K - A = K$

(K–4) If $\not\models A$ then $A \notin K - A$

(K–5) If $A \in K$ then $K \subseteq (K - A) + A$

(K–6) If $\models A \Leftrightarrow B$ then $K - A = K - B$

AGM Postulates for Composite Contraction

(K–7) $K - A \cap K - B \subseteq K - (A \wedge B)$

(K–8) If $A \notin K - (A \wedge B)$ then $K - (A \wedge B) \subseteq K - B$

Revision as Contraction and Expansion

Theorem: If a contraction function '–' satisfies (K–1) to (K–4) and (K–6), then the revision function '*' defined as

$$K * A = (K - \neg A) + A$$

satisfies (K*1) to (K*6). This is called the **Levi Identity**

Furthermore,

- if (K–7) is also satisfied, (K*7) will be satisfied
- if (K–8) is also satisfied, (K*8) will be satisfied

If we define contraction, this will also give us a revision function!

Contraction as Revision by the Negation

Theorem: If a revision function ‘*’ satisfies (K*1) to (K*6), then the contraction function ‘–’ defined as

$$K - A = K \cap K * \neg A$$

satisfies (K–1) to (K–6).

Furthermore,

- if (K*7) is also satisfied, (K–7) will be satisfied
- if (K*8) is also satisfied, (K–8) will be satisfied

Constructing Contraction

- A general idea is to start from K and then give some recipe for choosing which propositions to delete from K so that K – A does not contain A as a logical consequence.
- We should look for as large a subset of K as possible.
- A belief set K' is a **maximal subset** of K that fails to imply A if and only if
  1) $K' \subseteq K$
  2) $A \notin K'$
  3) For any sentence B that is in K but not in K', $B \Rightarrow A \in K'$
- The set of all belief subsets of K that fail to imply A is denoted K ⊥ A (also called the remainder set of K by A)

Selection Function and Maxichoice

- A first tentative solution to the problem of constructing a contraction function is to identify K – A with one of the maximal subsets in K ⊥ A
- Technically, this can be done with the aid of a selection function S
- S picks out an element S(K ⊥ A) of K ⊥ A for any K and any A whenever K ⊥ A is nonempty

(Maxichoice) K – A = S(K ⊥ A) when |≠ A, and K – A = K otherwise.

Any maxichoice contraction function satisfies (K–1) to (K–6), but they also satisfy the fullness condition

(K–F) If B ∈ K and B ∉ K – A, then B ⇒ A ∈ K – A for any belief set K.

Maximal Belief Set

- In a sense, maxichoice contraction functions in general produce contractions that are too large
- Let us say that a belief set K is **maximal** iff, for every sentence B, either B $\in$ K or ¬B $\in$ K

Theorem: If a revision function ‘*’ is defined from a maxichoice contraction function ‘–’ by means of the Levi identity, then, for any A such that ¬A $\in$ K, K * A will be maximal.

Full Meet Contraction

- The idea of full meet contraction is to assume that K – A contains only the propositions that are common to all of the maximal subsets in $K \perp A$

(Meet)

$$K - A = \begin{cases} \bigcap K \perp A, & K \perp A \neq \emptyset \\ K, & \text{otherwise.} \end{cases}$$

Any full meet contraction function satisfies (K–1) to (K–6), but they also satisfy the intersection condition

(K–I)

$$K - (A \wedge B) = (K - A) \cap (K - B)$$

Partial Meet Contraction

- The drawback of full meet contraction is that it results in contracted belief sets that are far too small.

Theorem: If a revision function ‘*’ is defined from a full meet contraction function ‘–’ by means of the Levi identity, then, for any A such that ¬A ∈ K, K * A = Cn({A}).

We can have the selection function S pick the “best” elements of K ⊥ A and then take their intersection:

(Partial meet) $K - A = \bigcap S(K \perp A)$

Transitively Relational Partial Meet Contraction

- What does “best” mean?
- We must be given a transitive and reflexive ordering relation ≤ on $K \perp A$
- Then we can define the selection function as follows

$$S(K \perp A) = \{K' \in K \perp A : \forall K'' \in K \perp A, K'' \leq K'\}$$

Theorem: For any belief set K, ‘–’ satisfies (K–1) – (K–8) iff ‘–’ is a transitively relational partial meet contraction function.

Computational Considerations

- Thus far, we have found a way of connecting the rationality postulates with a general way of modeling contraction functions
- The drawback of the partial meet construction is that the computational costs involved in determining what is in the relevant maximal subsets of a belief set K are so overwhelming that other solutions to the problem of constructing belief revisions and contractions should be considered.
- As a generalization of the AGM postulates several authors have suggested postulates for revisions and contractions of **bases** for belief sets rather than the belief sets themselves

Epistemic Entrenchment

- A second way of modeling contractions is based on the idea that some sentences in a belief system have a higher degree of **epistemic entrenchment** than others.
- The guiding idea for the construction of a contraction function is that when a belief set K is revised or contracted, the sentences in K that are given up are those having the **lowest degrees** of epistemic entrenchment.
- If A and B are sentences, the notation $A \leq B$ will be used as a shorthand for “B is at least as epistemically entrenched as A”.

Postulates for Epistemic Entrenchment

(EE1) If $A \leq B$ and $B \leq C$, then $A \leq C$	(transitivity)
(EE2) If $A \models B$, then $A \leq B$	(dominance)
(EE3) For any A and B, $A \leq A \wedge B$ or $B \leq A \wedge B$	(conjunctiveness)
(EE4) When $K \neq K_{\perp}$, $A \notin K$ iff $A \leq B$, for all B	(minimality)
(EE5) If $B \leq A$ for all B, then $\models A$	(maximality)

(C≤) $A \leq B$ if and only if $A \notin K - A \wedge B$ or $\models A \wedge B$.

(C–) $B \in K - A$ if and only if $B \in K$ and either $A < A \vee B$ or $\models A$.

Theorem: if ≤ satisfies (EE1) to (EE5), then the contraction uniquely determined by (C–) satisfies (K–1) to (K–8) as well as (C≤) and vice-versa

Thank you for your attention