Logic for AI — Master 1 Informatique
Class Assignment #10: Possibility Theory

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1 Possibility Measures

Let the universe Ω contain the following four interpretations:

$$\begin{aligned}\omega_0 &= \{P \mapsto 0, Q \mapsto 0\},\\ \omega_1 &= \{P \mapsto 0, Q \mapsto 1\},\\ \omega_2 &= \{P \mapsto 1, Q \mapsto 0\},\\ \omega_3 &= \{P \mapsto 1, Q \mapsto 1\}.\end{aligned}$$

Let us assume the possibility distribution π is given, such that

$$\begin{aligned}\pi(\omega_0) &= 0.3,\\ \pi(\omega_1) &= 1,\\ \pi(\omega_2) &= 0.1,\\ \pi(\omega_3) &= 0.2.\end{aligned}$$

Compute the following:

1. $\Pi(P)$;
2. $N(P)$;
3. $\Pi(P \Rightarrow Q)$;
4. $N(P \Rightarrow Q)$.

2 Semantics of a Possibilistic Belief Base

Let $\Sigma = \{(P \Rightarrow Q, 0.6), (P \vee R, 1), (\neg R, 0.2)\}$ be a possibilistic belief base. Compute its associated possibility distribution π_Σ.

3 Guaranteed Possibility

Given a possibility distribution π, a *guaranteed possibility measure*, noted Δ, is defined as:

$$\Delta(\phi) = \min_{\omega \models \phi} \pi(\omega).$$

In words, the guaranted possibility measure estimates to what extent *all* the models of ϕ are actually possible according to what is known, i.e., any model of ϕ is possible at least to degree $\Delta(\phi)$.

Derive the properties of Δ.