

# Logic for AI — Master 1 Informatique

## Class Assignment #8: Fuzzy Logic

Andrea G. B. Tettamanzi  
Université côte d’Azur  
andrea.tettamanzi@univ-cotedazur.fr

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### 1 Fuzzy Sets

Using your intuition and triangular, trapezoidal, or gaussian functions, define the membership functions on the real line for the following fuzzy sets:

1. the fuzzy number “approximately 3”;
2. the fuzzy number “approximately 2 or approximately 8”;
3. the fuzzy number “approximately 6 to approximately 8”;
4. for the weight of people: very light, light, average, heavy, very heavy;
5. for outdoor temperature: cold, cool, comfortable, warm, hot;
6. for alcohol content of a beverage: soft, light, hard.

### 2 Operations on Fuzzy Sets

Let

$$A = \frac{1}{a} + \frac{0.7}{b} + \frac{0.4}{c} + \frac{0.2}{d}$$
$$B = \frac{0.5}{b} + \frac{1}{c} + \frac{0.5}{d} + \frac{0.1}{e}$$

be fuzzy sets defined on the universe  $U = \{a, b, c, d, e\}$ . Compute (using the min t-norm and max t-conorm):

1.  $A \cup B$ ;
2.  $A \cap B$ ;
3.  $\overline{A}$ .
4.  $\overline{B}$ .
5.  $A \cap \overline{B}$ .
6.  $A \cap \overline{A}$ .

### 3 Extension Principle

Let  $f : U \times U \rightarrow \{0, 1, 2, 3, 4\}$ , where  $U$  is defined like in Exercise 2 and let  $f(x, y)$  be given by the following table:

| $x \setminus y$ | $a$ | $b$ | $c$ | $d$ | $e$ |
|-----------------|-----|-----|-----|-----|-----|
| $a$             | 0   | 1   | 2   | 3   | 4   |
| $b$             | 1   | 1   | 2   | 3   | 4   |
| $c$             | 2   | 2   | 2   | 3   | 4   |
| $d$             | 3   | 3   | 3   | 3   | 4   |
| $e$             | 4   | 4   | 4   | 4   | 4   |

Use the Extension Principle to compute the fuzzy set  $f(A, B)$ , where  $A$  and  $B$  are the sets defined in Exercise 2.

### 4 Fuzzy Inference

Consider the following rule base:

IF  $x$  is left AND  $v$  is neg THEN  $F$  is pos  
 IF  $x$  is left AND  $v$  is pos THEN  $F$  is zero  
 IF  $x$  is right AND  $v$  is neg THEN  $F$  is zero  
 IF  $x$  is right AND  $v$  is pos THEN  $F$  is neg

and let the linguistic values be defined as follows:

- for variable  $x$ :

$$\text{left}(x) = \begin{cases} 1, & x \leq -1; \\ (1 - x)/2, & -1 < x < 1; \\ 0, & x \geq 1; \end{cases} \quad \text{right}(x) = \begin{cases} 0, & x \leq -1; \\ (x + 1)/2, & -1 < x < 1; \\ 1, & x \geq 1; \end{cases}$$

- for variable  $v$ :

$$\text{neg}(v) = \begin{cases} 1, & v \leq -1; \\ (1 - v)/2, & -1 < v < 1; \\ 0, & v \geq 1; \end{cases} \quad \text{pos}(v) = \begin{cases} 0, & v \leq -1; \\ (v + 1)/2, & -1 < v < 1; \\ 1, & v \geq 1; \end{cases}$$

- for variable  $F$ :

$$\text{zero}(F) = \begin{cases} 1 - |F|, & |F| < 1; \\ 0, & |F| \geq 1; \end{cases}$$

$$\text{neg}(F) = \begin{cases} 1, & F \leq -1; \\ -F, & -1 < F < 0; \\ 0, & F \geq 0; \end{cases} \quad \text{pos}(F) = \begin{cases} 0, & F \leq 0; \\ F, & 0 < F < 1; \\ 1, & F \geq 1. \end{cases}$$

Compute the fuzzy set on  $F$  when  $x = -0.5$  and  $v = -0.1$ .