# Logic for AI — Master 1 IFI Class Assignment #2: Predicate Logic

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## 1 Formalization

Translate the following English sentences into predicate logic sentences:

- 1. Every person who owns an apartment pays the real estate tax.
- 2. A hotel is rated one-star if all rooms have shower and WC.
- 3. A person is a grandparent if they have a child who has children.

## 2 Nomenclature

Tell which qualifications among *an atom*, *a literal*, *ground*, *open*, and *closed* apply to the following sentences:

- 1.  $P(x) \lor Q(a)$ ,
- 2. R(x, y),
- 3.  $\neg \neg R(a, b)$ ,
- 4.  $\forall x \exists y R(x, y),$
- 5.  $P(a) \wedge R(a, b)$ ,
- 6.  $\neg P(a)$ ,
- 7.  $\neg P(x)$ ,
- 8.  $P(a) \wedge \exists x R(a, x)$ .

#### 3 Herbrand Base

Write the Herbrand base for the following languages:

- 1.  $\{a, P(\cdot), Q(\cdot)\},\$
- $2. \ \{0,1,\mathsf{Even}(\cdot),\mathsf{Lt}(\cdot,\cdot)\},$
- 3.  $\{a, b, f(\cdot), g(\cdot, \cdot), P(\cdot, \cdot, \cdot)\}.$

# 4 Herbrand Models

Given the Herbrand model  $\mathcal{I} = \{P(a), Q(b), R(a, b), S(a, a), S(b, b)\}$ , evaluate the following sentences:

- 1.  $\exists x R(a, x),$
- 2.  $\forall x \exists y R(x, y),$
- 3.  $\forall x P(x) \Rightarrow Q(x),$
- 4.  $\forall x S(x, x)$ .

# 5 Herbrand Entailment

Which of the following entailments hold, using Herbrand semantics?

- 1.  $Q(a) \models \forall x(P(x) \Rightarrow Q(x)),$
- 2.  $P(a) \models \neg Q(b) \land \exists x (P(x) \land Q(x)).$

## 6 Semantic Trees

Check the Herbrand satisfiability of the following sentence by applying Herbrand's Theorem:

$$\neg \left( \forall x P(x, a, g(x, b)) \Rightarrow \exists y \exists z P(f(y), z, g(f(a), b)) \right).$$