

Logic for AI - Class Assignment #6: Possibility Theory

Correction of the exercises

1. Possibility Measures

$$\begin{aligned}
 1.1) \quad \Pi(P) &= \max_{w \models P} \pi(w) = \text{[cf. slide 12]} && \text{models of } P: \\
 & && \{w_2, w_3\} \\
 &= \max \{ \pi(w_2), \pi(w_3) \} = \\
 &= \max \{ 0.1, 0.2 \} = 0.2
 \end{aligned}$$

$$\begin{aligned}
 1.2) \quad N(P) &= \min_{w \not\models P} \{ 1 - \pi(w) \} = \text{[cf. slide 12]} && \text{counter-models of } P: \\
 & && \{w_0, w_1\} \\
 &= \min \{ 1 - \pi(w_0), 1 - \pi(w_1) \} = \\
 &= \min \{ 1 - 0.3, 1 - 1 \} = \\
 &= \min \{ 0.7, 0 \} = 0
 \end{aligned}$$

$$\begin{aligned}
 1.3) \quad \Pi(P \Rightarrow Q) &= \max_{w \models P \Rightarrow Q} \pi(w) = \text{[cf. slide 12]} && (\neg P \vee Q) \\
 &= \max \{ \pi(w_0), \pi(w_1), \pi(w_3) \} = \text{models of } P \Rightarrow Q: \\
 &= \max \{ 0.3, 1, 0.2 \} = && \{w_0, w_1, w_3\} \\
 &= 1
 \end{aligned}$$

$$\begin{aligned}
 1.4) \quad N(P \Rightarrow Q) &= \min_{w \not\models P \Rightarrow Q} \{ 1 - \pi(w) \} = \text{[cf. slide 12]} \\
 &= \min \{ 1 - \pi(w_2) \} = \text{because } w_2 \text{ is the} \\
 &= \min \{ 1 - 0.1 \} = 0.9 && \text{only countermodel of } P \Rightarrow Q
 \end{aligned}$$

2. Semantics of a Possibilistic Belief Base

We are going to use the formula in slide 15. However, before we do that, we have to write down the interpretations in Ω for the formulas in Σ . Since they are constructed from 3 propositional variables P, Q, R , we have 8 interpretations:

	P	Q	R	$P \Rightarrow Q$	$P \vee R$	$\neg R$
w_0	0	0	0	1	0	1
w_1	0	0	1	1	1	0
w_2	0	1	0	1	0	1
w_3	0	1	1	1	1	0
w_4	1	0	0	0	1	1
w_5	1	0	1	0	1	0
w_6	1	1	0	1	1	1
w_7	1	1	1	1	1	0

← These three columns will come handy to determine whether a given $w \models \neg \phi$ in the formula: those will be the formulas for which we will find a 0 in the row corresponding to interpretation w !

