Logic for AI – Class Assignment A-7: Belief Revision

Correction of the exercises

1. Belief Sets

1.1) $Cn(\{P, Q\}) = \{P, Q, P \wedge Q, P \vee Q, P \Rightarrow Q, Q \Rightarrow P, \dots\}$ (where $P \Rightarrow Q = \neg P \vee Q$ and $Q \Rightarrow P = P \vee \neg Q$) + all the tautologies...

1.2) $Cn(\{P \vee Q, P \vee \neg Q\}) = \{P, P \vee Q, P \vee \neg Q, \dots\}$ + all the tautologies

2. Remainder Sets

2.1) $\{P, Q\} \perp\!\!\!\perp \{P \wedge Q\} = \{\{P\}, \{Q\}\}$

2.2) $\{P \vee R, P \vee \neg R, Q \vee S, Q \vee \neg S\} \perp\!\!\!\perp \{P \wedge Q\} =$

$\{\{P \vee R, Q \vee S, Q \vee \neg S\},$
$\{P \vee \neg R, Q \vee S, Q \vee \neg S\},$
$\{P \vee R, P \vee \neg R, Q \vee \neg S\},$
$\{P \vee R, P \vee \neg R, Q \vee S\}\}$

in other words, all the sets we can obtain by leaving out one of the four sentences, because $P \vee R$ and $P \vee \neg R$ are both required to halve P, and $Q \vee S$ and $Q \vee \neg S$ are both required to have Q.

3. AGM Postulates

3.1) We assume $A \in K$ and we prove (i) $K * A \subseteq K$ and (ii) $K \subseteq K * A$.

(i): by (K*3), $K * A \subseteq K + A$
but, since $A \in K$, $K + A = K$, therefore $K * A \subseteq K + A = K$;

(ii): we must assume $K \neq K_\perp$, otherwise there would be no reason to engage in belief revision (we revise beliefs in order to maintain consistency!); with this assumption at hand, it must be $\neg A \notin K$, for otherwise $K = K_\perp$, since $A \in K$! Therefore, (K*4) applies and $K + A \subseteq K * A$; but since $A \in K$, $K + A = K$; therefore, $K = K + A \subseteq K * A$.

Therefore, $K * A = K$, QED.

3.2) It is enough to show an example of K, A, B such that $(K * A) * B \neq (K * B) * A$.

We let $K = \{P, Q\}$, $A = Q$, $B = \neg Q$.

- $(K * A) * B = (\{P, Q\} * Q) * \neg Q = (\{P, Q\}) * \neg Q = \{P\}$
- $(K * B) * A = (\{P, Q\} * \neg Q) * Q = (\{P\}) + Q = \{P, Q\}$

Of course, $\{P\} \neq \{P, Q\}$.