BLIND IDENTIFICATION AND EQUALIZATION OF MIMO FIR CHANNELS BASED ON SECOND-ORDER STATISTICS AND BLIND SOURCE SEPARATION

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Abstract: In the single-user scenario of data communications, the identification and equalization of the channel can be accomplished blindly (i.e., without training sequences) using second-order statistics (SOS) if suitable diversity in the received signal is exploited. Diversity can be temporal, spatial, or both. In this paper one well known SOS-based method for blind identification and equalization (BIE) of communication channels is extended to the multi-input scenario. It is shown that the application of this SOS-BIE procedure reduces the system to a problem of blind source separation (BSS) of instantaneous linear mixtures. The users' signals can then be recovered in a second stage through appropriate BSS techniques, typically requiring higher-order statistics.

1. INTRODUCTION

1.1. Problem definition and objectives

In point-to-point digital communications, linear channel distortion introduces intersymbol interference (ISI) in the received signal, generating errors in symbol detection. Traditional equalizers were based on training sequences, while *blind* channel identification and equalization (BIE) methods do not require training sequences.

Original blind equalizers were based implicitly on the higher-order statistics (HOS) of the received signal [1, 2, 3]. However, due to the larger estimation error of HOS with respect to second-order statistics (SOS), these methods are computationally demanding. It is well known that when the input signal is stationary, SOS can only identify minimum-phase channels. The groundbreaking paper [4] first proved that nonminimum-phase channels *can* indeed be identified using SOS if the received signal exhibits cyclostationarity. SOS-based blind channel identification and equalization is possible in the so-called single-input multiple-input (SIMO) systems [4, 5]. In multiple-user communication environments, i.e., in multi-input multi-output (MIMO) scenarios, interference from other users — so-called multiple access interference (MAI) — adds to the ISI caused by multipath.

In this paper, we extend the method of [4] to the multi-user case. We prove that the direct extension of such procedure performs time equalization (i.e., removes ISI), but is unable to disentangle the spatial mixture of the users' signals. An instantaneous linear mixture of the transmitted data remains, which can be tackled (thus eliminating MAI) by means of suitable blind source separation techniques.

1.2. Notations

$\mathbb{C}$ is the set of complex numbers. $(A)_{ij}$ is the (i, j)-element of matrix A. I_n refers to the $n \times n$ identity matrix, and $0_{m\times n}$ to the matrix composed of $m \times n$ zeros; $\mathbf{0}_n$ is the n-zero column vector. Superindices $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{-1}$ and $(\cdot)^{\dagger}$ indicate the complex conjugate, transpose, Hermitian, inverse and Moore-Penrose pseudoinverse operators, respectively. $\mathrm{E}[\cdot]$ stands for mathematical expectation. Symbol δ_k denotes the discrete-time Dirac's delta function, whereas $\otimes$ represents the Kronecker product. If $X \in \mathbb{C}^{m\times n}$ and $Y \in \mathbb{C}^{p\times q}$, their Kronecker product $X \otimes Y \in \mathbb{C}^{mp\times nq}$ is given by

$$X \otimes Y = \begin{bmatrix} x_{11}Y & x_{12}Y & \dots & x_{1n}Y \\ x_{21}Y & x_{22}Y & \dots & x_{2n}Y \\ \vdots & \dots & \ddots & \vdots \\ x_{m1}Y & x_{m2} & \dots & x_{mn}Y \end{bmatrix}.$$

2. SIGNAL MODEL

2.1. Oversampling a single sensor

Consider a multi-user digital communication system composed of a single-sensor receiver in which:

(A1) q data sources simultaneously transmit mutually-independent information-bearing symbols $\{s_{k,m} \in \mathbb{C}\}$, $k = 1, \dots, q$, at a known rate $f_b = 1/T$ bauds. The information sequences fulfil: $\mathrm{E}[s_{k,\cdot}] = 0$ and $\mathrm{E}[|s_{k,\cdot}|^2] = 1$.

(A2) The impulse responses $h_k(t)$ representing the propagation between the kth source and the sensor (including the effects of the transmitter and receiver filters) has finite time span L_{h_k}. For simplicity, we will assume that $L_{h_k} = L_h$, $k = 1, \dots q$, spanning at most $M + 1$ data symbols, with $M = \lfloor L_{h_{\max}}/T \rfloor$.

(A3) The additive measurement noise $w(t)$ is zero-mean and uncorrelated with the data sequence.

With the above assumptions, the continuous-time complex baseband received signal can be expressed as:

$$x(t) = \sum_{k=1}^{q} \sum_{m=-\infty}^{\infty} s_{k,m} h_k(t - mT) + w(t). \qquad (1)$$

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Sampling at a rate $f_s = 1/T_s = P/T$ from an initial instant $t_0 = 0$ (without loss of generality) yields:

$$\begin{aligned} x(iT_s + nT) &= \sum_{k=1}^{q} \sum_{m=-\infty}^{\infty} s_{k,m} h_k\big(iT_s + (n-m)T\big) \\ &\quad + w(iT_s + nT) \\ &= \sum_{k=1}^{q} \sum_{m=0}^{M} s_{k,n-m} h_k(iT_s + mT) + w(iT_s + nT), \\ &\qquad i = 0, \ldots, P-1. \end{aligned} \quad (2)$$

Let us call $x_n^{(i)} = x(iT_s + nT)$, $h_{k,n}^{(i)} = h_k(iT_s + nT)$, and $w_n^{(i)} = w(iT_s + nT)$. Then:

$$x_n^{(i)} = \sum_{k=1}^{q} \sum_{m=0}^{M} s_{k,n-m} h_{k,m}^{(i)} + w_n^{(i)}. \quad (3)$$

Fractionally-spaced sampling effectively generates $L = P$ *virtual channels*. Symbol $h_{k,\cdot}^{(i)}$ denotes the discrete-time impulse response characterizing the transfer through the ith virtual channel between the kth source and the receiving sensor. Store N received samples of virtual channel i in vector $\mathbf{x}_n^{(i)} = [x_n^{(i)}, \ldots, x_{n-N+1}^{(i)}]^{\mathrm{T}}$. Parameter N is referred to as the smoothing factor [6] or stacking level [7]. Similarly, gather the N samples of the L virtual channel outputs in vector $\mathbf{x}_n = [\mathbf{x}_n^{(0)\mathrm{T}}, \ldots, \mathbf{x}_n^{(L-1)\mathrm{T}}]^{\mathrm{T}}$. With the notations $\mathbf{s}_{k,n} = [s_{k,n}, \ldots, s_{k,n-N-M+1}]^{\mathrm{T}}$, $\mathbf{s}_n = [\mathbf{s}_{1,n}^{\mathrm{T}}, \ldots, \mathbf{s}_{q,n}^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{h}_k^{(i)} = [h_{k,0}^{(i)}, \ldots, h_{k,M}^{(i)}]^{\mathrm{T}}$, $\mathbf{h}_k = [\mathbf{h}_k^{(0)\mathrm{T}}, \ldots, \mathbf{h}_k^{(L-1)\mathrm{T}}]^{\mathrm{T}}$

$$\mathbf{h} = [\mathbf{h}_1^{\mathrm{T}}, \ldots, \mathbf{h}_q^{\mathrm{T}}]^{\mathrm{T}} \quad (4)$$

$$H_{k,N}^{(i)} = \begin{bmatrix} h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} & \ldots & \ldots & 0 \\ 0 & h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} & \ldots & 0 \\ \vdots & & \ddots & & & \vdots \\ 0 & \ldots & 0 & h_{k,0}^{(i)} & \ldots & h_{k,M}^{(i)} \end{bmatrix}$$

$$H_{k,N} = [H_{k,N}^{(0)\mathrm{T}}, \ldots, H_{k,N}^{(L-1)\mathrm{T}}]^{\mathrm{T}} \quad (5)$$

$$H_N = [H_{1,N}, \ldots, H_{q,N}] \quad (6)$$

$$\mathbf{w}_n^{(i)} = [w_n^{(i)}, \ldots, w_{n-N+1}^{(i)}]^{\mathrm{T}} \quad (7)$$

$$\mathbf{w}_n = [\mathbf{w}_n^{(0)\mathrm{T}}, \ldots, \mathbf{w}_n^{(L-1)\mathrm{T}}]^{\mathrm{T}} \quad (8)$$

and relationship (3), we have the matrix model:

$$\mathbf{x}_n = H_N \mathbf{s}_n + \mathbf{w}_n. \quad (9)$$

Matrix $H_{k,N}^{(i)}$, with dimensions $N \times (N+M)$, is the so-called *filtering matrix* associated with the linear filter $\mathbf{h}_k^{(i)}$. Filtering matrix H_N, which represents the overall system, is of dimensions $LN \times q(N+M)$.

2.2. Multiple sensors

Alternatively, the receiver could exploit spatial diversity by means of K sensors located at different points in space. In such a case, let $h_k^{(i)}(t)$ denote the impulse response between the kth source and the ith sensor. The continuous-time received signal at the ith sensor then reads:

$$x^{(i)}(t) = \sum_{k=1}^{q} \sum_{m=-\infty}^{\infty} s_{k,m} h_k^{(i)}(t - mT) + w^{(i)}(t) \quad (10)$$

$i = 1, \ldots, K$. With obvious symbol redefinitions and suitable restatement of assumptions (A1)–(A3), the equivalent matrix model of the received signals sampled at the baud rate also shows the shape of (9). Now the number of virtual channels is equal to the number of sensors, $L = K$. In addition, one can oversample the output of each sensor by a factor of P, giving rise to $L = PK$ virtual channels. Sampling below the baud rate (*sub-sampling*) is also feasible as long as $L > q$ (as will be justified in the next section). In any case, all these situations can be represented by (9), a linear system of dimension $LN \times q(N+M)$. In the sequel, the subindex N (e.g., in H_N) will be dropped when no ambiguity exists.

2.3. Identifiability

The *objectives* of blind identification and equalization are to estimate H (blind channel identification) and $\mathbf{s}_n$ (blind channel equalization [ISI cancellation] and source separation [MAI cancellation]) from the only observation of the received vector $\mathbf{x}_n$. These tasks are tantamount to recovering the channel coefficient vector $\mathbf{h}$ [eqn. (4)] and the source vector

$$\begin{aligned} &\mathbf{s} = [s_{1,n}, \ldots, s_{q,n}]^{\mathrm{T}} = \\ &[\mathbf{s}_n(1), \mathbf{s}_n(N+M+1), .., \mathbf{s}_n((q-1)(N+M)+1)]^{\mathrm{T}} \end{aligned} \quad (11)$$

where $\mathbf{s}_n(i)$ denotes the ith element of vector $\mathbf{s}_n$.

A necessary condition for blind identifiability is that the filtering matrix be full column rank, which can only happen if $LN \geqslant q(N+M)$. This conditions fixes lower bounds on the sampling rate and/or the number of sensors. The sufficient conditions providing a filtering matrix $H_{k,N}$ with full column rank are [5]: 1) the polynomials $H_k^{(i)}(z) = \sum_{m=0}^{M} h_{k,m}^{(i)} z^m$ share no common zero, 2) N is greater than the maximum degree M of the polynomials $H_k^{(i)}(z)$, i.e., $N > M$, and 3) at least one polynomial $H_k^{(i)}(z)$ has degree M.

3. EXPLOITING SOURCE TEMPORAL WHITENESS

3.1. Introduction

Tong *et. al.* [4] were the first to realize that blind channel identification of nonminimum-phase systems is possible from the SOS alone in the single-user (SIMO) case. Their approach was based on the decomposition of the autocorrelation matrices of the observed vector at two different time lags. Here, we extend this approach to the multi-user scenario. The application of this extended procedure leaves an ISI-free spatial mixture of the sources unresolved. An extra assumption that the information sequence is temporally white is used in Tong's method. This assumption is easily extended to the multi-user case:

(A4) The autocorrelation function of symbol sequences is given by:

$$\mathrm{E}[s_{k,m}s_{k,n}^{*}] = \delta_{m-n}, \quad k = 1, \ldots, q. \tag{12}$$

Let us define the autocorrelation matrix of the observed vector as: $R_x(m) = \mathrm{E}[\mathbf{x}_n\mathbf{x}_{n-m}^{\mathrm{H}}]$. Accordingly, the autocorrelation matrix of the source vector process reads:

$$R_s(m) = \mathrm{E}[\mathbf{s}_n\mathbf{s}_{n-m}^{\mathrm{H}}] = \begin{cases} I_q \otimes J^m, & m \geqslant 0 \\ I_q \otimes (J^{\mathrm{H}})^{|m|}, & m < 0 \end{cases}$$

where J is the 'shifting' matrix

$$(J)_{ij} = \begin{cases} 1, & i = j+1 \\ 0, & \text{otherwise} \end{cases} \tag{13}$$

that is, a matrix of 1's along the first lower diagonal and 0's elsewhere. We also call $J_{\mathrm{B}} = I_q \otimes J$. From model (9), matrices R_x and R_s are related through: $R_x(m) = HR_s(m)H^{\mathrm{H}} + R_w(m)$, where $R_w(\cdot)$ is the autocorrelation matrix of the noise vector, defined accordingly. To simplify the following development, let us ignore the noise for the moment.

3.2. Channel identifiability

Theorem 1. *Suppose that H and $\mathbf{s}_n$ satisfy the linear model* (9) *and its constraints (A1)–(A4). Then H is determined from $R_x(0)$ and $R_x(1)$ up to a post-multiplicative factor of the form $Q \otimes I_{N+M}$, where $Q \in \mathbb{C}^{q\times q}$ is a $q \times q$ unitary matrix.*

Proof. Assume that the channel-source couples $(H, \mathbf{s}_n)$ and $(\tilde{H}, \tilde{\mathbf{s}}_n)$ fulfil the linear model (9) and constraints (A1)–(A4). Then: $R_x(0) = HH^{\mathrm{H}} = \tilde{H}\tilde{H}^{\mathrm{H}}$ and $R_x(1) = HJ_{\mathrm{B}}H^{\mathrm{H}} = \tilde{H}J_{\mathrm{B}}\tilde{H}^{\mathrm{H}}$. The first equality implies that $\tilde{H} = H\tilde{Q}$, with $\tilde{Q}$ a unitary matrix of dimensions $q(N+M) \times q(N \times M)$. The second equation implies that $J_{\mathrm{B}} = \tilde{Q}J_{\mathrm{B}}\tilde{Q}^{\mathrm{H}}$, which corresponds to the Jordan chain $J_{\mathrm{B}}\tilde{Q} = \tilde{Q}J_{\mathrm{B}}$. Let $\tilde{Q} = [\tilde{Q}^{(1)}, \ldots, \tilde{Q}^{(q)}]$, with $\tilde{\mathbf{q}}_i^{(k)}$ denoting the ith column vector of the kth $(N+M)$-column block $\tilde{Q}^{(k)} \in \mathbb{C}^{q(N+M)\times(N+M)}$, $k = 1, \ldots, q$, $i = 1, \ldots, N+M$. Due to the orthogonality of $\tilde{Q}$, we have

$$(\tilde{\mathbf{q}}_i^{(k)})^{\mathrm{H}}\tilde{\mathbf{q}}_j^{(l)} = \delta_{ijkl}. \tag{14}$$

On the one hand, $\tilde{Q}J_{\mathrm{B}} = [\tilde{Q}^{(1)}J, \ldots \tilde{Q}^{(q)}J]$, the lth column of $\tilde{Q}^{(k)}J$ being given by $\tilde{\mathbf{q}}_{l+1}^{(k)}$, for $l = 1, \ldots, N+M-1$, and $\mathbf{0}_{q(N+M)}$, for $l = N+M$. That is, for $k = 1, \ldots, q$:

$$J_{\mathrm{B}}\tilde{\mathbf{q}}_l^{(k)} = \tilde{\mathbf{q}}_{l+1}^{(k)}, \qquad l = 1, \ldots, N+M-1 \tag{15}$$

$$J_{\mathrm{B}}\tilde{\mathbf{q}}_{N+M}^{(k)} = \mathbf{0}_{q(N+M)}. \tag{16}$$

On the other hand, the lth column vector of the kth $(N+M)$-column block of $J_{\mathrm{B}}\tilde{Q}$ is equal to

$$J_{\mathrm{B}}\tilde{\mathbf{q}}_l^{(k)} = [0, \tilde{q}_{1,l}^{(k)}, \tilde{q}_{2,l}^{(k)}, \ldots, \tilde{q}_{N+M-1,l}^{(k)}, 0, \tilde{q}_{N+M+1,l}^{(k)}, \tilde{q}_{N+M+2,l}^{(k)}, \ldots, \tilde{q}_{2(N+M)-1,l}^{(k)}, \ldots]^{\mathrm{T}} \tag{17}$$

where $\tilde{q}_{i,j}^{(k)} = (\tilde{Q}^{(k)})_{ij}$. The combination of eqns. (16) and (17) shows that vector $\tilde{\mathbf{q}}_{N+M}^{(k)}$ is of the form

$$\tilde{\mathbf{q}}_{N+M}^{(k)} = [\mathbf{0}_{N+M-1}^{\mathrm{T}}, a_{1,k}, \ldots, \mathbf{0}_{N+M-1}^{\mathrm{T}}, a_{q,k}]^{\mathrm{T}} \tag{18}$$

with

$$\sum_{r=1}^{q}|a_{r,k}|^2 = 1 \tag{19}$$

by virtue of orthogonality relationship (14). Now, according to (15), $J_{\mathrm{B}}\tilde{\mathbf{q}}_{N+M-1}^{(k)} = \tilde{\mathbf{q}}_{N+M}^{(k)}$, which in combination with (17) gives: $\tilde{\mathbf{q}}_{N+M-1}^{(k)} =$

$$[\mathbf{0}_{N+M-2}^{\mathrm{T}}, a_{1,k}, b_{1,k}, \ldots, \mathbf{0}_{N+M-2}^{\mathrm{T}}, a_{q,k}, b_{q,k}]^{\mathrm{T}} \tag{20}$$

with $\sum_{r=1}^{q}(|a_{r,k}|^2 + |b_{r,k}|^2) = 1$. However, eqn. (19) implies that $b_{r,k} = 0$, $r = 1, \ldots, q$. A similar reasoning over the rest of the columns of $J_{\mathrm{B}}\tilde{Q}^{(k)}$ results in

$$\tilde{Q}^{(k)} = [a_{1,k}, \ldots, a_{q,k}]^{\mathrm{T}} \otimes I_{N+M} \tag{21}$$

which holds for $k = 1, \ldots, q$. As a result, we have that

$$\tilde{Q} = Q \otimes I_{N+M}, \qquad \text{with} \quad (Q)_{ij} = a_{i,j}. \tag{22}$$

Eqn. (14) implies that $\sum_{r=1}^{q} a_{r,i}^{*}a_{r,j} = 0$, $i \neq j$, which, along with (19), proves that Q is unitary: $Q^{\mathrm{H}}Q = I_q$. $\square$

3.3. Channel identification

The following is a direct extension of the blind channel identification procedure presented in [4] and explains how the channel can be estimated from $R_x(0)$ and $R_x(1)$. Let $R_x(0)$ have the singular value decomposition (SVD) $R_x(0) = US^2U^{\mathrm{H}}$ with $U \in \mathbb{C}^{LN\times q(N+M)}$ and $S = \mathrm{diag}(\sigma_1, \ldots \sigma_{q(N+M)})$. Since $R_x(0) = HH^{\mathrm{H}}$, $H = USV$, with $V \in \mathbb{C}^{q(N+M)\times q(N+M)}$ unitary.

Form the whitening matrix $W = S^{-1}U^{\mathrm{H}}$. Then

$$R = WR_x(1)W^{\mathrm{H}} = VJ_{\mathrm{B}}V^{\mathrm{H}} \tag{23}$$

which corresponds to the Jordan chain $RV = VJ_{\mathrm{B}}$. With $V = [V^{(1)}, \ldots, V^{(q)}]$, $\mathbf{v}_l^{(k)}$ representing the lth column of $V^{(k)}$, this chain can be decomposed into:

$$R\mathbf{v}_l^{(k)} = \begin{cases} \mathbf{v}_{l+1}^{(k)}, & l = 1, \ldots, N+M-1 \\ \mathbf{0}_{q(N+M)}, & l = N+M \end{cases}$$

for $k = 1, \ldots, q$. Assume R admits the SVD $R = U_R S_R V_R^{\mathrm{H}}$. Then

$$RR^{\mathrm{H}} = U_R S_R^2 U_R^{\mathrm{H}}. \tag{24}$$

Also, from (23)

$$RR^{\mathrm{H}} = VJ_{\mathrm{B}}J_{\mathrm{B}}^{\mathrm{H}}V^{\mathrm{H}} = VI_{\overline{0}\mathrm{B}}V^{\mathrm{H}} \tag{25}$$

with $I_{\overline{0}\mathrm{B}} = I_q \otimes I_{\overline{0}}$ and $I_{\overline{0}} = JJ^{\mathrm{H}} = \mathrm{diag}(0, \underbrace{1, 1, \ldots, 1}_{N+M-1})$.

From eqns. (24)–(25), it turns out that $\mathbf{v}_1^{(k)}$, $k = 1, \ldots, q$, can be identified as the left singular vectors associated with the zero singular values of matrix R. We can then construct

$$V^{(k)} = \left[\mathbf{v}_1^{(k)}, R\mathbf{v}_1^{(k)}, \ldots, R^{(N+M-1)}\mathbf{v}_1^{(k)}\right]. \tag{26}$$

Similarly,

$$R^{\mathrm{H}}R = V_R S_R^2 V_R^{\mathrm{H}} \tag{27}$$

$$R^{\mathrm{H}}R = VJ_{\mathrm{B}}^{\mathrm{H}}J_{\mathrm{B}}V^{\mathrm{H}} = VI_{\underline{0}\mathrm{B}}V^{\mathrm{H}} \tag{28}$$

with $I_{\underline{0}\mathrm{B}} = I_q \otimes I_{\underline{0}}$ and $I_{\underline{0}} = J^{\mathrm{H}} J = \mathrm{diag}(\underbrace{1, 1, \ldots, 1}_{N+M-1}, 0)$.

Hence, $\mathbf{v}_{N+M}^{(k)}$, $k = 1, \ldots, q$, can be identified from the right singular vectors associated with the zero singular values of matrix R. We can then construct

$$V^{(k)} = \big[(R^{\dagger})^{(N+M-1)}\mathbf{v}_{N+M}^{(k)}, \ldots, R^{\dagger}\mathbf{v}_{N+M}^{(k)}, \mathbf{v}_{N+M}^{(k)}\big].$$

Remark that there are indeed q different (left and right) singular vectors associated with the null singular vectors of R. Effectively, $\mathrm{rank}(R) = q(N+M) - q$, and hence $\dim\{\mathrm{Null}(R)\} = q$; in other words, the eigenspace associated with its zero eigenvalues is of dimension q. The orthogonality of V is guaranteed by the eigenstructure of R, which makes the columns of $V^{(k)}$ orthogonal and the matrix blocks $V^{(k)}$ possess mutually orthogonal columns.

3.4. Equalized outputs

By using the above procedure, the channel matrix is thus identified up to the indeterminacy shown in Theorem 1: if H represents the true channel, then the estimated channel will be $\hat{H} = H(Q \otimes I_{N+M})$, with Q an unknown $(q \times q)$ unitary matrix. Let $\mathbf{z}_n = \hat{H}^{\dagger}\mathbf{x}_n$ represent the equalized output vector. Then $\mathbf{z}_n = \hat{H}^{\dagger}H\mathbf{s}_n = (Q \otimes I_{N+M})^{\dagger}\mathbf{s}_n$. Also, $(Q \otimes I_{N+M})^{\dagger} = (Q \otimes I_{N+M})^{\mathrm{H}} = Q^{\mathrm{H}} \otimes I_{N+M}$. Hence, the relationship between the equalized outputs and the original data signals reads:

$$\mathbf{z}_n = (Q^{\mathrm{H}} \otimes I_{N+M})\,\mathbf{s}_n. \tag{29}$$

Now define $\mathbf{z} =$

$$[\mathbf{z}_n(1), \mathbf{z}_n(N+M+1), \ldots, \mathbf{z}_n\big((q-1)(N+M)+1\big)]^{\mathrm{T}} \tag{30}$$

where $\mathbf{z}_n(i)$ denotes the ith element of vector $\mathbf{z}_n$. With this notation, system (29) becomes

$$\mathbf{z} = Q^{\mathrm{H}}\mathbf{s} \tag{31}$$

where $\mathbf{s}$ is given by (11). In conclusion, the application of the extended Tong's method is able to eliminate ISI, but a unitary instantaneous linear mixture of the source data remains, as shown in eqn. (31). MAI elimination requires further processing in the form a BSS algorithm for instantaneous linear mixtures.

Remarks:

• If the source data is i.i.d. (as assumed by Tong's method — assumption A4), the separation of the spatial mixture requires a HOS-based BSS method, such as the well-known JADE [8] or ICA-HOEVD [9].

• The extended Tong's method resolves ISI and 'exhausts' all second-order information, including that of the spatial mixture, which then becomes unitary [10].

• The BSS problem has size $q \times q$, which is considerably reduced compared with $LN \times q(N+M)$, the original dimension of the BIE system (9).

• With the application of the extended Tong's method followed by BSS, we can perform channel equalization (ISI removal) and source separation (MAI removal) with a *single sensor* (providing the identification conditions set out in Section 2.3 are fulfilled). This joint space-time equalization is possible due to the diversity introduced by oversampling, which creates extra virtual sensors with additional linear mixtures of the signals.

• The extension to the case of known noise autocorrelation function can be done along the lines of [4].

4. CONCLUSIONS AND FURTHER WORK

In this contribution, we have extended the SOS-based blind FIR-channel identification method of [4] from the single-input case for which it was originally designed to the multi-input scenario. The solution provided by this direct extension eliminates ISI, but MAI removal requires a further processing stage composed of a suitable BSS method for instantaneous linear mixtures. The existence of this situation for subspace–based approaches in the multi–user environment was known. In this paper, we have seen that this is indeed the case for the particular blind identification method of [4]. Further work comprises the comparative performance analysis of the extended method relative to other blind identification techniques, such as that of [11], which is fully based on BSS.

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