Compressed sensing for the extraction of atrial fibrillation patterns from surface electrocardiograms

Amina Ghrissi and Vicente Zarzoso
Universite Côte d'Azur, CNRS, I3S Laboratory
Les Algorithmes, Euclide-B, 2000 route des Lucioles
06903 Sophia Antipolis Cedex, France
Email:{amina.ghrissi, vicente.zarzoso}@univ-cotedazur.fr

***Abstract*—The non invasive analysis of atrial fibrillation (AF) arrhythmia represents a challenge nowadays. The fibrillatory pattern of AF, known as f-wave, is partially masked by the ventricular activity of the heartbeat in the surface electrocardiogram (ECG). Classical techniques aiming to extract the f-wave are based on average beat subtraction (ABS) or blind source separation (BSS). They present limitations in performance and require long ECG records as well as multi-channel records in the case of BSS. The originality of the present work consists in exploiting the sparsity of the atrial activity (AA) signal in the frequency domain to extract the full f-wave using a recent data acquisition technique called compressed sensing (CS). The present contribution takes a step forward in the extraction of the f-wave by exploiting the time rather than the space dimension. We intend to recover the AA signal with a variant of CS where classical random sampling is replaced by a block sampling scheme. Our breakthrough finding consists in the ability of our method to accurately extract the AA from a short ECG record of just one heartbeat, with a normalized mean squared error of 15%, which is unfeasible with ABS, BSS and other variants that require longer observation windows.**

I. Introduction

Atrial fibrillation (AF) is the most common sustained arrhythmia encountered in clinical practice. Held responsible of up to 25% of strokes, this cardiac condition is considered as the last great frontier of cardiac electrophysiology as it continues to puzzle cardiologists [1]. In order to better characterize this arrhythmia, scientists are interested in analyzing the pattern of AF noninvasively by extracting the f-waves of atrial activity (AA) from surface electrocardiogram (ECG) records [2]. The main classical cardiac signal processing tools for non invasive AA signal extraction are 1) average beat subtraction (ABS) [3], and 2) blind source separation (BSS) [4], [5]. To provide adequate performance, these techniques require records of sufficient length. Other techniques like interpolation are also adapted to AA extraction [6].

The present work aims at overcoming the limitations of ABS and BSS. We intend to extract the AA and separate it from the dominating ventricular activity (VA, QRST complex) using compressed sensing (CS). This method takes advantage of the *sparsity* property of the fibrillatory signal in the frequency domain. To our knowledge, this is the first time CS is applied to noninvasive AA extraction. Our second contribution consists in introducing a *block sampling* scheme as opposed to the random sampling classically used in CS. We start by preliminary tests on simple signals in order to validate the implementation of our solution and its accuracy. The tests focus on the influence of the *compression ratio* on the quality of signal recovery added to the effect of block sampling. Second, CS is computed on synthetic f-waves. Third, we move to more realistic scenarios by applying CS to synthetic ECG signals with fibrillatory pattern. We compare CS performance to a state of the art ABS technique, adaptive singular value cancellation (ASVC) [3]. Comparison is mainly based on the quality of AA extraction, sensitivity to ECG record length and computational cost.

This work is outlined as follows. Section II presents the signal extraction problem and summarizes classical cardiac signal processing tools used for f-wave estimation. Section III introduces CS in the context of AF ECG analysis. We study its mathematical definition and properties. Our results of CS signal reconstruction with different sampling approaches are presented in Section IV. Finally, the conclusions and perspectives of the work are summarized in Section V.

II. Noninvasive extraction of AA in AF

A. AF Diagnosis

AF occurs when there exists an irregular and chaotic activation in the atrias, the upper chambers of the heart. Then instead of beating effectively to eject blood into the ventricles, the atria start quivering or fibrillating, thus causing irregular fluctuations in the baseline. As a result, the ventricular rate becomes more rapid and irregular.

The ECG of patients suffering from AF is different from normal sinus rhythm. It is characterized by the absence of P-wave and the presence of f-waves visible in the TQ segments between consecutive beats. A TQ segment is the time interval between the end of a T-wave (end of a heartbeat) and the beginning of the next Q-wave (beginning of the next heartbeat), whereases a QT segment corresponds to the interval between the beginning and the end of a given heartbeat. The AA occurs all throughout the recoding, but it is obscured by VA at each heartbeat [2]. Indeed, in the ECG we distinguish two kinds of intervals:

1) TQ segments: only AA takes place and it is perfectly known since the ventricles are inactive.

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2) QT segments: both AA and VA happen simultaneously but the AA is masked by the QRST complex.

Our goal is estimating the AA in the QT segments in AF patients.

B. Atrial Activity Extraction

We aim to extract the AA and separate it from the VA in ECG recordings. Several methods have been proposed for this task like ABS and BSS. Although partially successful, the latter methods present important limitations, as summarized below:

a) ABS: Although multi-channel variants exist, ABS is mainly adapted to single-lead ECGs [3]. This method computes a representative beat by synchronized averaging of the beats present in the recordings, thus requiring the recognition of beat morphology from the ECG. ABS is very sensitive to QRST wave variants and relies on high-quality cancellation templates that are in practice difficult to obtain from short single-lead ECG recordings.

ASVC of ventricular activity is a variant of ABS, developed in [3], that intends to overcome ABS inherent limitations. The method exploits the mutual information available in the set of ECG beats in order to extract the basis signal corresponding to the VA component. ASVC remains more robust than ABS in ECGs with variable QRST morphology and in the presence of ectopic beats. However, ASVC is limited by the number of beats to be processed and the length of ECG signals.

b) BSS: BSS is a statistical tool that consists in separating unobservable source signals from a set of observed mixture. Independent component analysis (ICA) belongs to BSS and is proven in [4] to accurately recover AA in AF. ICA reconstructs the unobservable independent sources of bioelectric activity which generate, through instantaneous linear mixing, a measurable set of signals. This approach exploits the spatial diversity provided by multi-lead ECG recordings with sufficient length to allow the estimation of higher order statistics with enough accuracy.

III. Proposed Approach

A. Essence of Compressed Sensing

Compressed sensing (CS) is a data compression paradigm that requires much less measurements than imposed by the Nyquist rate. CS acquires a spread/dense signal f accepting a sparse/compressible representation x, when expressed in the proper basis $\mathbf{\Psi}$:

$$f = \mathbf{\Psi} x. \tag{1}$$

In practice we only observe a subset $\Omega \subset \{1, .., N\}$ of size $|\Omega| = K \ll N$, through a selection matrix $\mathbf{\Phi}$ and the challenge is to recover the N-dimensional sparse signal x from y the K-dimensional measurement vector:

$$y = \mathbf{\Phi} f = \mathbf{\Phi}\mathbf{\Psi} x = \mathbf{U} x, \tag{2}$$

$\mathbf{U}$ is a $(K \times N)$ matrix. The sampling scheme is generally random.

CS recovers the sparse representation x through suitable optimization tools [7]. The symbol $\|.\|_1$ stands for ℓ_1 norm:

$$\min_{x \in R^N} \|x\|_1 \quad \text{subject to} \quad y = \mathbf{U} x. \tag{3}$$

From a linear algebra perspective, the reconstruction problem is ill posed, because $K < N$. CS guarantees the accurate reconstruction of the target signal through conditions and problem regularizations like the *Restrictive Isomery Property* (RIP) [7], [8].

B. Compressed Sampling for Noninvasive Atrial Activity Extraction from the ECG

Based on the physiological finding that the ventricular and atrial activities are uncoupled [1] and that the ECG is the sum of both, added to interference from the activities of the surrounding organs, vessels and noise due to the acquisition system, we intend to extract the full AA from the observation of its signal in the TQ segment. In this context, we suggest to restrict the sample selection on the TQ segments from the ECG, where VA is null. We call this sampling scheme *block sampling*. The f-wave (f_{AA}) is present all along the ECG recording (f_{ECG}). The component f_{AA} accepts a sparse representation in the frequency domain as in Eq. (1). f_{VA} is the interfering component present in QT intervals of ECG. f_{AA} is present in both heart beating phases QT (f_{ECG}^{QT}) and TQ (f_{ECG}^{TQ}):

$$f_{ECG} = f_{AA} + f_{VA}, \tag{4}$$

$$f_{ECG}^{QT} = f_{AA}^{QT} + f_{VA}^{QT}, \tag{5}$$

$$f_{ECG}^{TQ} = f_{AA}^{TQ}. \tag{6}$$

The measurement signal is $y = f_{ECG}^{TQ} = f_{AA}^{TQ}$. The CS approach aims at reconstructing the full f_{AA}, including the unknown f_{AA}^{QT}, from the only knowledge of y. We intend to use the samples of AA in TQ segments (f_{AA}^{TQ}), which are perfectly known, to estimate/reconstruct, through CS, the unknown samples in QT intervals (f_{AA}^{QT}). Because the theoretical demonstration of this approach is a difficult task, the goal is to experimentally evaluate the block sampling scheme for AA signal recovery.

TABLE I
BLOCK SAMPLING OF ECG.

Compressed sampling algorithm for AA extraction from the ECG
Input: ECG signal of length N, f_{ECG}
Output: $\hat{f}_{AA}$
1 **CS matrix computation:**
2 **Block Sampling**: select blocks of TQ intervals from ECG to form the measurement signal y, of size $K < N$
3 **Recovery**: reconstruct optimal $\hat{x}$.
4 **Estimation**: $\hat{f}_{AA} = F^{-1}(\hat{x})$, F is Fourier transform matrix

Block Sampling: Classical sampling process takes K *random* measurements. Our proposed sampling scheme consists

in taking blocks of the measurement of size (p), corresponding to $f_{ECG}^{TQ} = f_{AA}^{TQ}$, alternated by blocks of unobserved signals f_{ECG}^{QT} of size (q), as illustrated in Fig. 1. We call this scheme (p, q) sampling. In practice, we measure the TQ and drop the QT intervals.

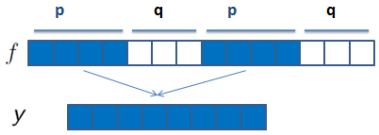

Fig. 1. Observed signal formation with *(p,q)* sampling scheme.

Our suggestion for block sampling needs to be experimentally validated and mathematically proven. We find in the literature a prior work that neglected parts of signal (image rows/columns) in the sampling, when applying CS to ultrasound images in [9]. This technique showed accurate signal reconstruction even under high compression ratios.

IV. Experimental results

This section investigates the performance of CS technique for atrial signal (f_{AA}) recovery in the case of patients with AF. In particular, interest is focused on the influence of block as opposed to random sampling in CS, as we intend to recover the f_{AA} from measures restricted to the TQ segment in ECG recordings. Our solution is validated and tested on synthetic ECG signals containing AF patterns.

Complexity is added at each experiment level. The influence of the compression ratio on the accuracy of signal recovery is analyzed. In order to quantify the recovery quality, the normalized mean squared error (NMSE) between the original and the reconstructed signals is computed. Then, signal is sampled by equal-sized-blocks of measurement instead of the standard random sampling protocol (block-sampling). First experiments are conducted on synthetic f-wave signals (f_{AA} with AF pattern) generated according to the model of Stridh [11]. Second experiments are conducted on synthetic f_{AA} superimposed to real ECG signals of VA (f_{VA}) of subjects in sinus rhythm. The resulting signal approximates the ECG of an AF patient. Third, the CS approach is applied to a complete ECG with AF patterns available in an open source database. Finally, the performance of CS is compared to that of ASVC in AA extraction from ECG. The experiments on CS are conducted with MATLAB software using the ℓ_1-MAGIC toolbox [10].

A. CS Recovery of isolated f-wave

1) Genesis of Synthetic f-wave: We simulate synthetic f-wave according to Stridh model [11]. The dynamics of AF have a modulated sawtooth-like shape, approximated by a sinusoid and $(M-1)$ harmonics. The sawtooth amplitude is given by a time-varying amplitude a_i, a phase $\theta(n)$ and cycle length, thus introducing a non-stationary behavior:

$$f(n) = \sum_{i=1}^{M} a_i(n) sin(i\theta(n)), \qquad n = 1, .., N. \tag{7}$$

We recall θ depends on the fundamental frequency of this pattern, typically in the range of $[3, 9]$ Hz. We simulate a paroxysmal AF pattern with $M = 5$ harmonics, main amplitude $a = 150\ \mu V$ (a_i depends on a) and dominating frequency of 6 Hz as in [11].

2) Influence of Block Size and CR on the NMSE: We perform bare AF signal recovery ($f = f_{AA}$) for different combinations of p and q yielding the compression ratio (CR):

$$\mathrm{CR} = \frac{p}{p+q}. \tag{8}$$

After sampling, the recovery is assessed by the NMSE between the original signal and the recovered one:

$$\mathrm{NMSE}(f) = \frac{\|f - \hat{f}\|_2^{\,2}}{\|f\|_2^{\,2}}. \tag{9}$$

Fig. 2 shows the curves of the NMSE versus different combinations of p and CR in logarithmic scale. The experiments are conducted on a 6-second recording.

Assuming typical durations of QT and TQ segments in normal heart rhythm [12], the CR is approximated by the ratio $\frac{\mathrm{TQ}}{\mathrm{TQ+QT}}$:

$$\mathrm{CR} \simeq \frac{\mathrm{TQ}}{\mathrm{TQ+QT}} \in [0.4, 0.6]. \tag{10}$$

For CR $\in [0.4, 0.6]$, the NMSE resulting from block sampling, for all values of p are in $[-10, 0]$ (dB). For values of CR close to 0.9 and 1, the NMSE curves tend to $-\infty$ in (dB).

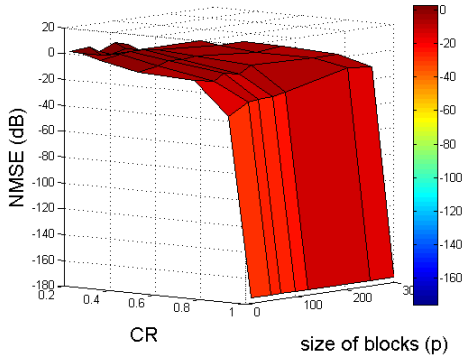

Fig. 2. NMSE (dB) of reconstructed synthetic atrial signal for block sampling for different combinations of p (number of samples) and CR.

B. CS Recovery of f-wave Corrupted by VA

1) Genesis of Synthetic ECG with Fibrillatory Pattern: Similarly to the process of data synthesis in [1], we suggest to generate synthetic ECG contaminated by AF pattern by superimposing synthetic f-waves generated by the above Stridh model [11] to a VA signal containing only QT complex. The f_{VA} is generated from a surface ECG of a healthy subject after the following steps:

1) Acquire sinus rhythm surface ECG data of healthy adults from the PTB Diagnostic ECG Database (PTB) [13], at a sampling rate of 1 KHz. 2) Denoise the data and remove

baseline wander and powerline interference using a forward-backward bandpass type-II Chebyshev IIR filter with cut-off frequencies of 0.5 Hz and 30 Hz. 3) Manually delete P-waves as they explicitly reflect the AA in sinus rhythm. The P-waves are segmented and then suppressed by spline interpolation between their onset and offset points.

We focus our experiments on ECG signals acquired from the precordial $V1$ lead because the AA is significantly clearer and measurable in the chest position where this lead is placed.

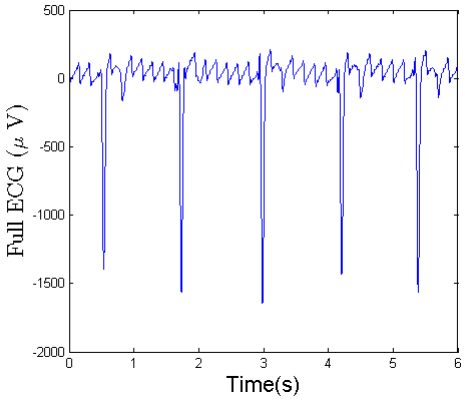

Fig. 3. Synthetic ECG with AF pattern.

C. Extraction of f-wave from Full Synthetic ECG

1) Long recording: Fig. 4 shows that both algorithms, CS and ASVC succeed in estimating a fibrillatory pattern in both locations of QT segments. However, ASVC outperforms CS with an NMSE of f_{AA} recovery in TQ segments (f_{AA}^{TQ}) equal to 0.406 compared to 1.4. Accuracy is also better when recovering non measured segments NMSE(f_{AA}^{QT}). The CS algorithm running time takes 0.5 h compared to 1.05 s in the case of ASVC.

Recovery of isolated f-wave: We use the same sampling scheme as previously explained to recover the isolated f-wave using the CS solution. The overall recovery quality is acceptable with an NMSE(f_{AA}) = 0.201. Also, the recovery error is below machine precision in the sampled data (TQ segments).

However, ASVC computes a synchronous mean of the heart beats then subtracts the average QRST complex from ECG. For this purpose this techniques operates on recordings having full QRST complexes.

2) Short recording: We have seen in the previous experiment that the ASVC outperforms CS and that its computational cost is significantly less important due to the complexity of solving an optimization convex program in CS compared to synchronously averaging beats in ASVC. However, these experiments also lead us to the following observations:

One-Heartbeat ECG: CS is able to recover the signal from a recordingas short as 1 s, containing 1 heartbeat in average, as illustrated in Fig. 6. However, ASVC is not designed to work in this case, as its performance is said to be acceptable for an ECG recording containing at least 10 beats [3], that is, almost 15 s (15000 samples for a high resolution ECG sampling frequency f_s = 1 Khz).

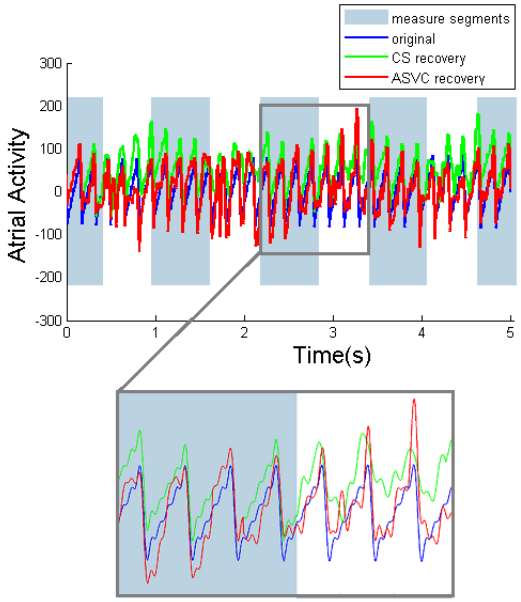

Fig. 4. Extraction of f-wave from synthetic ECG using CS (green) and ASVC (red) vs. the original fibrillatory signal (blue).

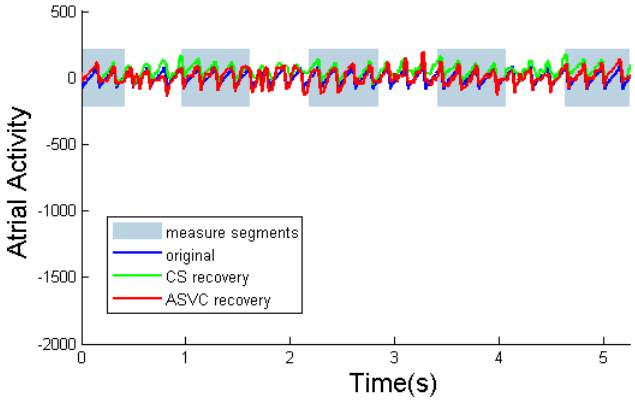

Fig. 5. Extraction of f-wave from synthetic ECG using CS (green) and ASVC (red) vs. the original fibrillatory signal (blue).

Bias-corrected CS or Corrected CS (CCS): We notice in the previous experiment (One-Heartbeat ECG) that although the recovered AA with CS mimics almost perfectly the original one, its NMSE is important (NMSE(f_{AA}) = 1.797). When observing Fig. 6, we notice the $\hat{f}_{AA}$ seems biased by a trend but its overall shape is accurate. To suppress the bias, we suggest to subtract from the restriction of $\hat{f}_{AA}$ to each segment ($\hat{f}_{AA}^{\text{segment}_i}$) its mean, where $(\text{segments}_i)_{i=1..3}$ represent the samples of intervals TQ and QT alternatively. We notice discontinuity between consecutive segments because their means are different. To handle this issue, we suggest to correct the CS technique by subtracting from $\hat{f}_{AA}$ its

full mean (mean($\hat{f}_{AA}$)). The overall NMSE(f_{AA}) after bias correction equals just 0.150. Hence, the performance of CS

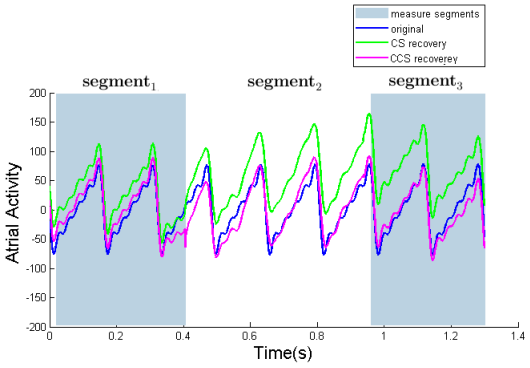

Fig. 6. Extraction of f-wave from synthetic ECG (1-heartbeat recording) with bias-corrected CS (magenta) vs. standard CS (green) vs. the original fibrillatory signal (blue).

TABLE II
NMSE OF f-WAVE EXTRACTION FROM SYNTHETIC AF ECG WITH BIAS-CORRECTED CS FROM A 1-HEARBEAT ECG.

Algorithm	NMSE($f_{AA}^{\text{segment}_1}$)	NMSE($f_{AA}^{\text{segment}_2}$)	NMSE($f_{AA}^{\text{segment}_3}$)
CCS	0.127	0.194	0.115

is improved by mean subtraction for each TQ and QT segment.

V. CONCLUSIONS

The present contribution has put forward a new technique for extracting the fibrillatory pattern of AF from surface ECG based on the CS paradigm. To our knowledge, this is the first time CS is applied to AA extraction. We suggest to exploit the fact that the AA in the ECG has a sparse frequency distribution and therefore it can be recovered using CS. We propose to replace the classical random sampling of the measurement signal in CS by a block sampling that selects only VA-free segments from ECG and neglects QRS complexes, thus recovering the full AA.

The results show that the accuracy of CS is lower than ASVC in terms of NMSE when processing full long recordings. However, a breakthrough finding is the ability of CS to extract AA from a short ECG recording containing only one heartbeat, which is impossible with ASVC, which is said to perform well for significantly longer recordings of at least 10 heartbeats. Based on the observation that CS performs well on short recordings, it appears well suited to online processing, where the AA is estimated beat-by-beat from the ECG. In this manner, the CS approach may handle better long recordings. On the other hand, ASVC needs to perform heartbeats detection and classification before the extraction process, thus being very sensitive to heartbeat morphology, location and duration. A bias-corrected variant of our method proves to be more accurate with an NMSE(f_{AA}) equal 0.150 versus 1.797 in classical CS technique. Finally, we asses the influence of the number of heartbeats on the accuracy of CS recovery. A major drawback of CS is its high computational cost. Despite these apparent limitations, experimental results are encouraging. Further work should aim at justifying mathematically the validity of block sampling according to the RIP and validating our method on a full AF ECG database. The sensitivity of our approach to heartbeat morphology should also be verified.

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