# Blind Channel Identification in $(2 \times 1)$ Alamouti Coded Systems Based on Maximizing the Eigenvalue Spread of Cumulant Matrices

Héctor J. Pérez-Iglesias[1], Daniel Iglesia[1], Adriana Dapena[1], and Vicente Zarzoso[2]

[1] Departamento de Electrónica y Sistemas, Universidade da Coruña, Facultad de Informática, Campus de Elviña, 5, 15071 A Coruña, Spain
{hperez,dani,adriana}@udc.es
[2] Laboratoire I3S, Université de Nice-Sophia Antipolis, Les Algorithmes, Euclide-B, BP 121, 06903 Sophia Antipolis, France
zarzoso@i3s.unice.fr

**Abstract.** Channel estimation in the $(2\times1)$ Alamouti space-time block coded systems can be performed blindly from the eigendecomposition (or diagonalization) of matrices composed of the receive antenna output 4th-order cumulants. In order to estimate the channel, we will propose to choose the cumulant matrix with maximum eigenvalue spread of cumulant-matrix. This matrix is determined in closed form. Simulation results show that the novel blind channel identification technique presents a satisfactory performance and low complexity.

## 1 Introduction

A large number of Space Time Coding (STC) techniques have been proposed in the literature to exploit spatial diversity in systems with multiple elements at both transmission and reception (see, for instance, [1] and references therein). The Orthogonal Space Time Block Coding (OSTBC) is remarkable in that it is able to provide full diversity gain with linear decoding complexity [2,3]. The basic premise of OSTBC is the encoding of the transmitting symbols into an unitary matrix to spatially decouple their Maximum Likelihood (ML) detection, which can be seen as a matched filter followed by a symbol-by-symbol detector.

The OSTBC scheme for MIMO systems with two transmit antennas is known as the Alamouti code [2] and it is the only OSTBC capable of achieving full spatial rate for complex constellations. The $(2 \times 1)$ Alamouti coded systems are attractive in wireless communications due to their simplicity and their ability to provide maximum diversity gain while preserving the channel capacity. Because of these advantages, the Alamouti code has been incorporated in the IEEE 802.11 and IEEE 802.16 standards [4].

Coherent detection in $(2 \times 1)$ Alamouti coded systems requires the identification of a $(2\times2)$ unitary channel matrix. The transmission of data known to the receiver, known as pilot or training symbols, is often used to perform

T. Adali et al. (Eds.): ICA 2009, LNCS 5441, pp. 694–701, 2009.

channel estimation required for a coherent detection of OSTBCs. However, training symbols reduce the throughput and such schemes are inadequate when the bandwidth is scarce. Strategies that avoid this limitation include the so-called Differential STBC (DSTBC) [5] which is a signalling technique that generalizes differential modulations to the transmission over MIMO channels. DSTBCs can be incoherently decoded without the aid of channel estimates but they incur in a 3 dB performance penalty when compared to coherent detection.

Training sequences can also be avoided by the use of blind channel identification methods. In particular, this contribution focuses on blind channel identification in $(2 \times 1)$ Alamouti coded systems using higher-order eigen-based approaches. Under the assumption of independent symbol substreams, the channel can be estimated from the eigendecomposition of matrices composed of 2nd- or higher-order statistics (cumulants) of the received signal. The so-called joint approximate diagonalization of eigenmatrices (JADE) method for blind source separation via independent component analysis is optimal in that it tries to simultaneously diagonalize a full set of 4th-order cumulant matrices. In order to reduce the computational complexity, we propose to diagonalize a linear combination of cumulant matrices, which is judiciously chosen by maximizing its expected eigenvalue spread.

## 2 The $(2 \times 1)$ Alamouti Coding Scheme

Figure 1 shows the baseband representation of Alamouti OSTBC with two antennas at the transmitter and one antenna at the receiver. Each pair of symbols $\{s_1, s_2\}$ is transmitted in two adjacent periods using a simple strategy: in the first period $s_1$ and $s_2$ are transmitted from the first and the second antenna, respectively, and in the second period $-s_2^*$ is transmitted from the first antenna and $s_1^*$ from the second one, symbol $(\cdot)^*$ denoting complex conjugation. In the sequel, we assume that the symbol substreams are complex-valued, zero-mean, stationary, non-Gaussian distributed and statistically independent; their exact probability density functions are otherwise unknown.

The transmitted symbols (sources) arrive at the receiving antenna through fading paths $h_1$ and $h_2$ from the first and second transmit antenna, respectively. Hence, the signal received during the first and the second symbol period have the

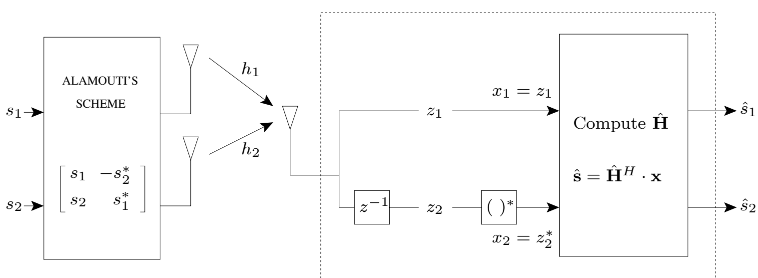


**Fig. 1.** Alamouti Coding Scheme

form $z_1 = s_1 h_1 + s_2 h_2 + n_1$ and $z_2 = s_1^* h_2 - s_2^* h_1 + n_2$, respectively. The term $n_i$ denotes the additive white Gaussian noise at symbol period $i$. By defining the observation vector as $\mathbf{x} = [x_1, x_2]^{\mathrm{T}} = [z_1, z_2^*]^{\mathrm{T}}$, symbol $(\cdot)^{\mathrm{T}}$ standing for the transpose operator, the relationship between the observation vector $\mathbf{x}$ and the source vector $\mathbf{s} = [s_1, s_2]^{\mathrm{T}}$ is given by

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{n} = [n_1, n_2^*]^{\mathrm{T}}$ is the noise vector and $\mathbf{H}$ represents the $(2 \times 2)$ channel matrix

$$\mathbf{H} = \left[\, \mathbf{h}_1 \middle| \mathbf{h}_2 \,\right] = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \tag{2}$$

It is interesting to note that matrix $\mathbf{H}$ is unitary up to a scalar factor, i.e.,

$$\mathbf{H}\mathbf{H}^{\mathrm{H}} = \mathbf{H}^{\mathrm{H}}\mathbf{H} = \|\mathbf{h}\|^2 \mathbf{I}_2 \tag{3}$$

where $\|\mathbf{h}\|^2 = |h_1|^2 + |h_2|^2$ is the squared Euclidean norm of the channel vector, $\mathbf{I}_2$ is the $(2 \times 2)$ identity matrix and $(\cdot)^{\mathrm{H}}$ is the Hermitian operator. It follows that the transmitted symbols can be recovered, up to scale, as $\hat{\mathbf{s}} = \hat{\mathbf{H}}^{\mathrm{H}}\mathbf{x}$, where $\hat{\mathbf{H}}$ is a suitable estimate of the channel matrix. As a result, this scheme supports ML detection based only on linear processing at the receiver. Consequently, the correct detection of the transmitted symbols $\mathbf{s}$ requires the accurate estimation of the channel matrix $\mathbf{H}$ from the received data $\mathbf{x}$.

## 3 Blind Channel Estimation Based on Eigenvalue Spread

In this section, we will propose a novel higher-order eigen-based approach to estimate the channel matrix in the $(2 \times 1)$ Alamouti system. For the sake of simplicity, we restrict the exposition to zero-mean distributions and circular statistics. Given a random vector $\mathbf{x} = [x_1, x_2] \in \mathbb{C}^2$, its 2nd-order cumulants are simply defined as $\mathrm{cum}(x_i, x_j^*) = \mathrm{E}[x_i x_j^*]$, and the 4th-order cumulants as

$$\mathrm{cum}(x_i, x_j^*, x_k, x_\ell^*) =$$

$$= \mathrm{E}[x_i, x_j^*, x_k, x_\ell^*] - \mathrm{E}[x_i x_j^*]\mathrm{E}[x_k x_\ell^*] - \mathrm{E}[x_i x_\ell^*]\mathrm{E}[x_j x_k^*] - \mathrm{E}[x_i x_k]\mathrm{E}[x_j^* x_\ell^*] \tag{4}$$

Given a matrix $\mathbf{M} \in \mathbb{C}^{2\times 2}$

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} \tag{5}$$

the 4th-order cumulant matrix $\mathbf{Q}^{(4)}(\mathbf{M})$ is defined as the $(2 \times 2)$ matrix with components [6]

$$[\mathbf{Q}^{(4)}(\mathbf{M})]_{ij} = \sum_{k,\ell=1}^{2} \mathrm{cum}(x_i, x_j^*, x_k, x_\ell^*) m_{\ell k} \tag{6}$$

Under a linear model like (1) with statistically independent sources and unitary mixing matrix, the cumulant matrix takes the form $\mathbf{Q}^{(4)}(\mathbf{M}) = \mathbf{H}\boldsymbol{\Delta}(\mathbf{M})\mathbf{H}^{\mathrm{H}}$. Matrix $\boldsymbol{\Delta}(\mathbf{M})$ being diagonal with

$$[\boldsymbol{\Delta}(\mathbf{M})]_{ii} = \rho_i \mathbf{h}_i^{\mathrm{H}} \mathbf{M} \mathbf{h}_i \tag{7}$$

where $\rho_i = \mathrm{cum}(s_i, s_i^*, s_i, s_i^*)$ is the marginal 4th-order cumulant (kurtosis) of the $i$th source, and $\mathbf{h}_i$ represents the $i$th column of $\mathbf{H}$. Therefore, the separating matrix $\mathbf{H}$ diagonalizes $\mathbf{Q}^{(4)}(\mathbf{M})$ for any $\mathbf{M}$. Hence, the eigendecomposition of (6) allows the identification of the remaining unitary part of $\mathbf{H}$ if the eigenvalues of $\mathbf{Q}^{(4)}(\mathbf{M})$ are different, i.e., if matrix $\boldsymbol{\Delta}(\mathbf{M})$ contains different entries: $\rho_i \mathbf{h}_i^{\mathrm{H}} \mathbf{M} \mathbf{h}_i \neq \rho_j \mathbf{h}_j^{\mathrm{H}} \mathbf{M} \mathbf{h}_j$, $\forall i \neq j$. To increase robustness to eigenspectrum degeneracy, a set $\{\mathbf{Q}^{(4)}(\mathbf{M}_k)\}_{k=1}^{m}$, may be (approximately) jointly diagonalized. The full set comprises $m = 2^2 = 4$ linearly independent (e.g., orthonormal) matrices $\{\mathbf{M}_k\}_{k=1}^{m}$. A simplified version of the algorithm is obtained by considering the set of matrices verifying $\mathbf{Q}^{(4)}(\mathbf{M}_k) = \lambda_k \mathbf{M}_k$. As there are only 2 such eigenmatrices, this version is, in theory, computationally more efficient. However, the eigenmatrices depend on matrix $\mathbf{H}$ itself and they must also be estimated from the data. JADE admits an efficient implementation in terms of the Jacobi technique for matrix diagonalization. In Alamouti Coding Scheme, the channel matrix is unitary, and can then be identified by this procedure.

### 3.1 Maximizing the Eigenvalue Spread

The performance of eigendecomposition-based methods depend on the eigenvalue spread of the matrix to diagonalize because the eigenvectors associated with equal eigenvalues can only be determined up to a unitary transformation [7]. For the $(2 \times 1)$ Alamouti OSTBC, with the same kurtosis for $s_1$ and $s_2$, $\rho = \rho_1 = \rho_2$, from equation (7) the eigenvalue spread of cumulant matrix $\mathbf{Q}^{(4)}(\mathbf{M})$ is

$$L(\mathbf{M}) = |\rho| \left| \mathbf{h}_1^{\mathrm{H}} \mathbf{M} \mathbf{h}_1 - \mathbf{h}_2^{\mathrm{H}} \mathbf{M} \mathbf{h}_2 \right| \tag{8}$$

In order to obtain the matrix $\mathbf{M}_{opt}$ that maximizes $L(\mathbf{M})$ we will introduce the following notation,

$$\tilde{\mathbf{h}} = \begin{bmatrix} |h_1|^2 - |h_2|^2 \\ 2h_1^* h_2^* \\ 2h_1 h_2 \\ |h_2|^2 - |h_1|^2 \end{bmatrix}, \quad \mathbf{m} = \begin{bmatrix} m_{11} \\ m_{21} \\ m_{12} \\ m_{22} \end{bmatrix} \tag{9}$$

Substituting in (8), the eigenvalue spread takes the form

$$L(\mathbf{M}) = |\rho||\mathbf{h}_1^H \mathbf{M} \mathbf{h}_1 - \mathbf{h}_2^H \mathbf{M} \mathbf{h}_2| =$$

$$|\rho||(|h_1|^2 - |h_2|^2)(m_{11} - m_{22}) + 2(h_1 h_2 m_{21} + h_1^* h_2^* m_{12})| = |\rho||\tilde{\mathbf{h}}^{\mathbf{H}} \mathbf{m}| \tag{10}$$

Our objective is to find the vector $\mathbf{m}$ that maximizes the eigenvalue spread, i.e.,

$$\mathbf{m}_{opt} = \begin{matrix} arg\ max \\ ||\mathbf{m}|| = 1 \end{matrix} \quad |\tilde{\mathbf{h}}^H \mathbf{m}| \tag{11}$$

The scalar product between $\mathbf{h}$ and $\mathbf{m}$ is maximum when $\mathbf{m}$ has the direction and sense of $\mathbf{h}$. Therefore, the optimum value is

$$\mathbf{m}_{opt} = \frac{\tilde{\mathbf{h}}}{||\tilde{\mathbf{h}}||} \tag{12}$$

As a result, the optimum matrix is given by

$$\mathbf{M}_{opt} = \frac{1}{\sqrt{2+2|\gamma|^2}} \begin{bmatrix} 1 & \gamma \\ \gamma^* & -1 \end{bmatrix}, \text{where} \quad \gamma = \frac{2h_1 h_2}{|h_1|^2 - |h_2|^2} \tag{13}$$

Since this optimum matrix depends on the actual values of unknown channel coefficients $h_1$, $h_2$, we propose to estimate parameter $\gamma$ using 4th-order cross-cumulants of the observations,

$$\gamma = \frac{\text{cum}(x_1, x_2^*, x_1, x_2^*)}{\text{cum}(x_1, x_2^*, x_1, x_1^*)} = \frac{\rho 2 h_1^2 h_2^2}{\rho(|h_1|^2 - |h_2|^2) h_1 h_2} = \frac{2h_1 h_2}{|h_1|^2 - |h_2|^2} \tag{14}$$

Note that the parameter $\gamma$ is used to weighted 4th-order cross-cumulants in equation (6) and, therefore, errors in its estimation can degrade the performance of the method proposed to identify the channel matrix $\mathbf{H}$.

### 3.2 Blind Channel Estimation Based on Eigenvalue Spread (BCEES)

To evaluate the the 4th-order cumulant function (6) in the optimum matrix $\mathbf{M}$ given in equation (13) requires to compute sixteen 4th-order cross-cumulant. According to the symmetric property of the cumulants, it can be reduced to compute six fourth-order cross-cumulants. In this subsection, we will propose a simplified approach which only consider to fourth-order cross cumulant matrices:

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{M}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \tag{15}$$

For these two matrices, the cumulant sum given in equation (6) is reduced to the computation of only one 4th-order cumulant matrix. Particularizing equation (10) to these matrices, we can determine which of the two matrices provides the maximum eigenvalue spread using to the following decision criterion

$$|\gamma| = \frac{L(\mathbf{M}_2)}{L(\mathbf{M}_1)} = \frac{2|h_1||h_2|}{||h_1|^2 - |h_2|^2|} = \frac{2|h_1||h_2|}{||h_1|^2 - |h_2|^2|} \underset{\mathbf{M}_2}{\overset{\mathbf{M}_1}{\lessgtr}} 1 \tag{16}$$

In the practice, $\gamma$ can be estimated using (14).

## 4 Experimental Results

This section reports several numerical experiments carried out to evaluate and compare the performance of the blind channel estimation algorithms proposed

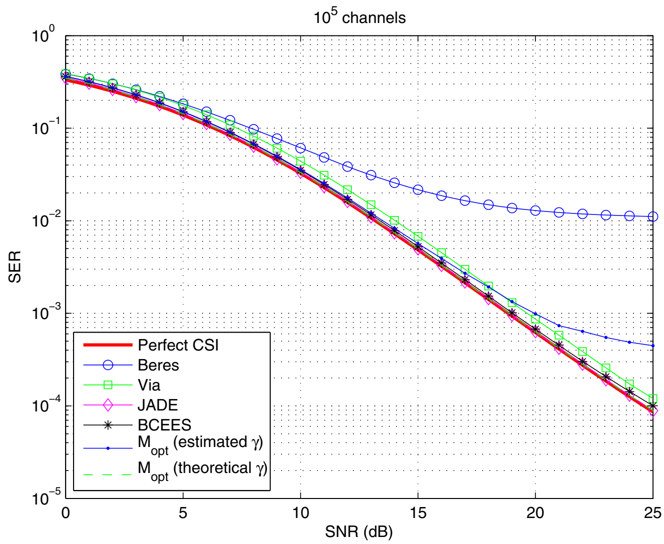


**Fig. 2.** SER versus SNR for Rayleigh channels for a block size of 500 symbols

in this paper. The experiments have been performed on QPSK source symbols coded with the Alamouti Scheme. The channel is assumed to remain constant during the transmission of a block of $K$ symbols and it has a Rayleigh distribution. The cumulants are calculated by sample averaging over the symbols of a block. Performance has been measured in terms of the Symbol Error Rate (SER) of the source symbols estimated after channel identification. SER figures are obtained by averaging over $10^5$ independent blocks of symbols and channel realizations.

Over this simulated scenario, the following techniques are compared:

- The method proposed by Beres *et al.* [8] corresponding to use $m_{11} = 1, m_{12} = m_{21} = m_{22} = 0$ or $m_{11} = 0, m_{12} = m_{21} = m_{22} = 2$.
- The SOS-based approach proposed by Via *et al.* [9].
- The JADE algorithm proposed by Cardoso and Souloumiac [6].
- The novel method based on maximizing the eigenvalue spread proposed in Subsection 3.1 using the theoretical $\gamma$.
- The novel method based on maximizing the eigenvalue spread proposed in Subsection 3.1 using the estimated $\gamma$ in equation (14).
- The BCEES technique proposed in Subsection 3.2, using the estimated $\gamma$ in equation (14).

As a bound of performance, we also present the SER obtained with Perfect Channel Side Information (CSI).

Figure 2 shows the performance obtained with these methods for different values of SNR. The 4th-order cumulants have been estimated using 500 symbols of each observed signals. Note that the results obtained with the novel approaches and JADE are very close to the Perfect CSI. Note, however, that for high SNRs

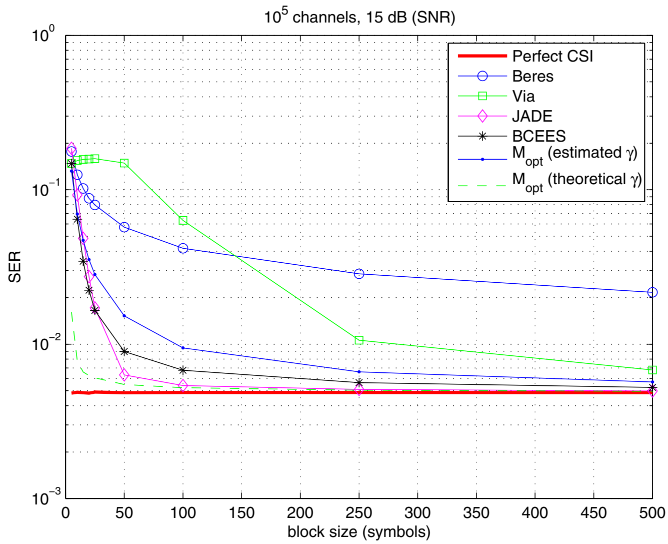


**Fig. 3.** SER versus packet size for Rayleigh channels

the novel method with the estimated $\gamma$ presents a flooring effect due to errors in the estimation of $\gamma$.

Figure 3 shows the performance for different packet sizes at an SNR of 15 dB. The good performance of JADE, the novel approach with theoretical $\gamma$ and BCEES is apparent.

The decoding complexity of methods based on cumulant-matrix diagonalization depends on two parameters: the number of cumulant matrices to be computed and the size of the matrix to be diagonalized. Note that the load of diagonalizing $2 \times 2$ matrices is very low because it can be used closed expressions [10]. Table 1 summarizes these parameters for the approaches considered in our simulations. Considering the computational load and the results presented in Figure 2 and Figure 3, we conclude that BCEES presents a satisfactory performance and low complexity.

**Table 1.** Computational load

| Approach | Number of cumulant matrices | Size of the matrix to diagonalize |
|---|---|---|
| Beres et al., and Via et al. | 1 | $2 \times 2$ |
| JADE | 4 | $3 \times 3$ |
| $\mathbf{M}_{opt}$ | 4 | $2 \times 2$ |
| BCEES | 2 | $2 \times 2$ |

## 5 Conclusion

This paper addresses the problem of blind channel identification in $(2 \times 1)$ Alamouti coded system. In order to estimate the channel matrix, we have proposed to diagonalize a linear combination of cumulant matrices, which is judiciously chosen by maximizing its expected eigenvalue spread. We have also proposed a simplified approach, called BCEES, which presents a satisfactory performance and low complexity since it needs to diagonalize a single cumulant matrix.

## References

1. Paulraj, A.J., Papadias, C.B.: Space-time processing for wireless communications. IEEE Signal Processing Magazine 14(6), 49–83 (1997)
2. Alamouti, S.M.: A simple transmit diversity technique for wireless communications. IEEE Journal Select. Areas Communications 16, 1451–1458 (1998)
3. Tarokh, V., Jafarkhani, H., Calderbank, A.R.: Space-Time Block Codes from Orthogonal Designs. IEEE Transactions on Information Theory 45(5), 1456–1467 (1999)
4. Andrews, J.G.: Fundamentals of WiMAX: Understanding Broadband Wireless Networking. Prentice Hall Communications Engineering and Emerging Technologies Series (2007)
5. Hughes, B.L.: Differential space-time modulation. IEEE Transactions on Information Theory 46(7), 2567–2578 (2000)
6. Cardoso, J.-F., Souloumiac, A.: Blind beamforming for non-Gaussian signals. IEE Proceedings F 140(46), 362–370 (1993)
7. Golub, G.H., Van Loan, C.F.: Matrix Computations, 3rd edn. Johns Hopkins Studies in Mathematical Sciences (1996)
8. Beres, E., Adve, R.: Blind Channel Estimation for Orthogonal STBC in MISO Systems. In: Proc. of Global Telecommunications Conference, vol. 4, pp. 2323–2328 (November 2004)
9. Vía, J., Santamaría, I., Pérez, J., Ramírez, D.: Blind Decoding of MISO-OSTBC Systems based on Principal Component Analysis. In: Proc. of International Conference on Acoustic, Speech and Signal Processing, vol. IV, pp. 545–549 (May 2006)
10. Blinn, J.: Consider the lowly 2x2 matrix. Computer Graphics and Applications 16(2), 82–88 (1996)