ALPHABET-BASED DEFLATION FOR BLIND SOURCE EXTRACTION IN UNDERDETERMINED MIXTURES

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ABSTRACT

The deflation approach to blind source extraction estimates the source signals one by one. The contribution of the latest source estimate is computed via linear regression and subtracted from the observations before performing a new extraction. In the context of digital communications, novel alphabet-based contrast criteria can naturally be defined, leading to the recently proposed parallel deflation concept. We analyse the use of such criteria in the challenging scenario of underdetermined mixtures, where the sources outnumber the sensors. Due to the limitations of linear extraction, projection on the signal alphabet before the regression-subtraction stage is shown to be capital for a successful source estimation. It is also demonstrated that alphabet-based criteria outperform the constant modulus (CM) principle, even for CM-type sources. More interestingly, classical deflation can improve on parallel deflation, but requires a refinement to render its performance robust to the extraction ordering.

Keywords: alphabet-based criteria, blind source separation, deflation, digital communications, MIMO transmission, underdetermined mixtures.

1 INTRODUCTION

The goal of blind source separation (BSS) is to recover the unknown source signals from their observed mixtures. The deflation approach to BSS consists of estimating the source signals one after another. Originally proposed by Delfosse-Loubaton [3] in the context of instantaneous linear mixtures, deflation was later applied with success by Tugnait in the convolutive scenario [10]. After estimating a single source signal using a suitable cost or contrast function, its contribution to the sensor output is estimated (via linear regression, for instance) and subtracted from the observations. The process is then repeated until all sources have been extracted. In multiple-input multiple-output (MIMO) digital communications, deflation (or symbol cancellation) has also been employed by the popular V-BLAST detection algorithm [5], which requires an accurate channel matrix estimate and is thus non-blind.

Despite its appealing simplicity, deflation presents two main drawbacks. Firstly, estimation errors caused in each extraction-deflation stage accumulate through successive stages. As a result, the source estimation quality deteriorates progressively as more sources are obtained. Secondly, since a linear extractor is usually employed, the maximum number of sources that can be separated is limited by the available spatial diversity, i.e., it is generally impossible to extract more sources than sensors. This limits the applicability of deflation in the interesting scenario of underdetermined mixtures.

The discrete nature of digital modulation sources, characterized by a finite number of symbols composing the signal alphabet or constellation, can help alleviate these shortcomings. The present contribution analyses and compares these alphabet-exploiting techniques for deflation-based source extraction in underdetermined mixtures. A simple modification improves the robustness of classical deflation to the source extraction ordering, and outperforms the recently proposed parallel deflation [8] in estimating all sources with low error probability more often.

2 PROBLEM AND ASSUMPTIONS

A noisy mixture $\mathbf{x} = [x_1, x_2, \ldots, x_L]^{\mathrm{T}} \in \mathbb{C}^L$ of K uncorrelated sources $\mathbf{s} = [s_1, s_2, \ldots, s_K]^{\mathrm{T}} \in \mathbb{C}^K$ is observed at the output of an L-sensor array, where $^{\mathrm{T}}$ denotes transposition. In matrix form, the sensor output can be expressed as:

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n} = \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathbf{n} \qquad (1)$$

where $\mathbf{H} \in \mathbb{C}^{L \times K}$ represents the unknown full-rank mixing matrix with columns $\{\mathbf{h}_k\}_{k=1}^{K}$, and $\mathbf{n} \in \mathbb{C}^L$ the additive noise, which is also unknown, uncorrelated with the sources, and has covariance matrix $\sigma_n^2 \mathbf{I}_L$. Eqn. (1) models (but is not limited to) a flat-fading MIMO transmission system. BSS aims at estimating the realizations of random

In *Proc. ICA Research Network International Workshop*, Liverpool, UK, Sept. 18-19, 2006, pp. 21-24.

vector $\mathbf{s}$ from the observation of the corresponding realizations of the mixture $\mathbf{x}$. To this end, we seek an extracting vector $\mathbf{w} \in \mathbb{C}^L$ so that the linear extractor output

$$y = \mathbf{w}^{\mathrm{H}}\mathbf{x} \tag{2}$$

optimises some cost function or contrast criterion. Symbol $^{\mathrm{H}}$ represents the Hermitian (conjugate-transpose) operator. After a source signal has been estimated in this fashion, its contribution is computed and subtracted (cancelled) from the observations, which then become 'deflated'. The source estimation and deflation process is repeated until all signals have been extracted.

In the challenging underdetermined mixture scenario, the number of sources is higher than the number of sensors, $K > L$. In that case, it is generally not possible to estimate all sources linearly, even in the absence of noise, as the rows of the mixing matrix only span an L-dimensional subspace of $\mathbb{C}^K$. Similarly, linear extraction severely limits the capabilities of conventional deflation, as will be seen later, calling for the design of alternative extraction and/or deflation criteria.

The novelty of the present approach lies in the exploitation of the discrete character of digital communication signals. In the sequel, it will be assumed that the sources can be divided into R different groups, $\sum_{r=1}^{R} K_r = K$, where group r contains K_r sources with the same digital modulation $\mathcal{A}_r$. Each digital modulation is characterized by its alphabet or constellation $\mathcal{A}_r = \{a_{r,m}\}_{m=1}^{M}$, whose discrete symbols can be represented by the roots of a polynomial $\psi_r(z) = \prod_{m=1}^{M}(z - a_{r,m})$.

3 ALPHABET-BASED EXTRACTION

Under the signal model and assumptions of the previous section, it follows that functional

$$\mathcal{J}_r(y) = \mathrm{E}\{|\psi_r(y)|^2\}$$

is a contrast function for sources with alphabet $\mathcal{A}_r$ under rather general assumptions [2]. In particular, a constellation may not be a subset of another. This criterion, originally proposed in [6], is known as alphabet polynomial fitting (APF) and becomes the so-called constant power (CP) criterion for M-PSK modulations [11]. The APF presents the advantage of targeting a specific signal modulation, in contrast to alternative criteria typically used in the separation of digital communication sources such as the constant modulus (CM) or the kurtosis maximisation (KM) principles [4, 9]. As opposed to independence-based contrast criteria, the APF can separate spatially correlated and spectrally coloured sources. To estimate a source signal of given modulation, a simple yet efficient gradient-descent procedure with optimal step size can drive a linear extractor in the search of the corresponding APF contrast-function minima [11].

4 CLASSICAL DEFLATION

At the end of a successful iterative search (leading to the optimisation of the corresponding contrast function $\mathcal{J}_r$), the extractor output y contains an estimate $\hat{s}$ of a source signal s with alphabet $\mathcal{A}_r$. In regression-based classical deflation, the contribution of the extracted source to the observations is estimated as:

$$\hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \mathrm{E}\{\|\mathbf{x} - \mathbf{h}\hat{s}\|^2\} \;\Rightarrow\; \hat{\mathbf{h}} = \frac{\mathrm{E}\{\mathbf{x}\hat{s}^*\}}{\mathrm{E}\{|\hat{s}|^2\}} \tag{3}$$

symbol * denoting complex conjugation, and then subtracted to yield the deflated sensor output:

$$\mathbf{x} \;\leftarrow\; \mathbf{x} - \hat{\mathbf{h}}\hat{s}. \tag{4}$$

If a linear extractor is employed, as in eqn. (2), it is easy to prove that the rank of the sensor-output covariance matrix (related to the available spatial diversity) necessarily decreases by one at each deflation step, regardless of the achieved source estimation quality. As a result, only L out of the K sources can at most be estimated by this procedure. This fundamental limitation renders plain classical deflation inappropriate in the underdetermined case.

5 ALPHABET-BASED DEFLATION

5.1 Parallel Deflation

Estimation errors accumulate through successive stages in classical deflation. Parallel deflation [8] tries to overcome this limitation by exploiting the discrete nature of digital sources and alphabet diversity, which arises when $R > 1$. Sources from alphabet $\mathcal{A}_r$ are extracted using the corresponding APF criterion. To minimise the impact of error accumulation, the deflation process used for the sources with a given modulation is carried out from the original observations, that can be processed in parallel by the appropriate APF contrasts. As a result, one such parallel deflation processes 'perceives' a mixture of K_r sources on L sensors, which should be easier to deal with than the L mixtures of K sources 'seen' by conventional deflation over all sources. Nevertheless, the extraction of sources from group r may be severely hampered by the interfering sources from the other groups.

5.2 Projection on the Source Alphabet

As pointed out earlier, the linear estimate of a source signal reduces the rank of the deflated sensor-output covariance matrix, making it impossible to extract all sources in an underdetermined mixture. To circumvent this difficulty, let us assume that the source has been perfectly estimated: $\hat{s} = s_k$, for some $k \in \{1, \dots, K\}$. Then, under the source uncorrelation assumption, the deflation procedure described by eqns. (3)–(4) would produce $\hat{\mathbf{h}} = \mathbf{h}_k$ and the new set of observations $\mathbf{x} = \sum_{p \neq k} \mathbf{h}_p s_p + \mathbf{n}$; that is, the interference caused by that source to the remaining sources would be perfectly cancelled. Since the rank of the deflated sensor-output covariance matrix would not necessarily decrease, the rest of the sources might still be extracted at later stages.

Obviously, it will generally be difficult to have $\hat{s} = s_k$. A simple manner to try to obtain this perfect estimate is by projecting the linear extractor output on the known source constellation before deflation, as in the V-BLAST detection algorithm [5]. This non-linear processing can be carried out cost-effectively by the minimum-distance detector.

5.3 Optimal Ordering in Classical Deflation

Classical deflation reduces the remaining interference as more sources are extracted. The amount of interference reduction depends on the quality of the source estimate. To minimise error accumulation, the 'strongest' or best estimated sources should be extracted and deflated first. The prior knowledge of the channel matrix simplifies the optimal ordering in terms of the output signal-to-noise ration (SNR), as in the V-BLAST algorithm [5]. For the blind scenario, we propose the following ordering method which, for simplicity but without loss of generality, is developed for M-PSK modulations.

The symbol error probability in the detection of a M-PSK signal contaminated by complex Gaussian noise can be accurately approximated by [7]:

$$P_e = 2Q\left(\sqrt{\mathrm{SNR}}\sin\left(\frac{\pi}{M}\right)\right), \quad M > 2 \qquad (5)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \mathrm{e}^{-t^2/2}dt$. Now, given the set $\{\hat{s}_p, \hat{\mathbf{h}}_p, \mathbf{w}_p\}_{p=1}^K$ provided by an initial deflation sweep, the signal-to-interference-and-noise ratio (SINR) in the estimation of source k can be computed as:

$$\mathrm{SINR}_k = \frac{|\mathbf{w}_k^{\mathrm{H}}\hat{\mathbf{h}}_k|^2}{\sum_{p\neq k}|\mathbf{w}_k^{\mathrm{H}}\hat{\mathbf{h}}_p|^2 + \hat{\sigma}_n^2\|\mathbf{w}_k\|^2}. \qquad (6)$$

The noise variance estimate $\hat{\sigma}_n^2$ can be obtained from the sensor-output residual after all sources have been deflated. To estimate the probability of error P_e in (5), the SNR can be replaced with the SINR given above. Deflation can then be repeated in ascending order of P_e or, equivalently, descending order of $\sqrt{\mathrm{SINR}}\sin(\pi/M)$. To target a specific source while trying to alleviate the increased computational cost, the linear extractor found in the original deflation is used to initialise the iterative optimisation of the corresponding alphabet-matched contrast function (the CP criterion for M-PSK signals). The whole process may be repeated until the ordering converges, or just for a fixed number of deflation iterations.

6 EXPERIMENTAL STUDY

Influence of extraction criterion and alphabet projection. An underdetermined instantaneous linear mixture of 4 sources with QPSK modulation is observed at the output of a 3-sensor array in blocks of 150 data symbols. The sensor output is corrupted by additive white complex circular Gaussian noise, with a varying spatially averaged received SNR defined as in [5], which can be expressed as $\mathrm{SNR} = \mathrm{trace}(\mathbf{H}\mathbf{H}^{\mathrm{H}})/(L\sigma_n^2)$. The mixing matrix elements are randomly drawn from a normalised complex Gaussian distribution at each of the 200 Monte Carlo iterations. In the first experiment, two extraction criteria (CM and CP) together with two deflation methods (classical deflation and classical deflation with projection), giving rise to the methods labelled as CM-D, CM-P-D (projection on $|s| = 1$), CP-D and CP-P-D (projection on the alphabet). The search for the CM and the CP contrast function minima is carried out with the optimal step-size technique of [11]. ZF V-BLAST with perfect channel estimate is implemented as in [5]. The linear MMSE detector and the non-linear MAP detector serve as performance bounds.

Figure 1 (top) shows the symbol-error-rate (SER) averaged over the 4 estimated sources. Figure 1(bottom) displays the probability of extracting all 4 sources with an SER below 10%. The CM-D and the CP-D, where deflation is based on conventional linear regression, are unable to extract the four sources satisfactorily. Likewise, the MMSE extractor and V-BLAST are also severely limited by the lack of linear invertibility of the channel matrix. Although the CM-P-D visibly improves on the CM-D, the combination of alphabet projection and alphabet-based extraction appears most effective. Indeed, the CP-P-D approaches the MAP bound and, for sufficient SNR, is able to extract all four sources at low SER with probability close to one.

Classical vs. parallel deflation. Influence of extraction ordering. The second experiment simulates a mixture of 6 sources, three with BPSK and three with 3-PSK modulation, observed at the output of 4 sensors, in the same general conditions as above and 150 Monte Carlo iterations. Only CP-based extraction is considered: classical deflation with direct ordering (targeting the BPSK sources first), with inverse ordering (aiming at the 3-PSK sources first), and with the optimal ordering presented in the previous section (with a single extra deflation sweep after ordering). These methods are also compared with the parallel deflation approach of [8] with alphabet projection.

As observed in Fig. 2, the performance of classical deflation depends strongly on the extraction ordering, with the proposed optimal ordering achieving the best results at almost twice the average number of optimal step-size gradient-descent iterations (around 550) required by the two other classical deflation methods (300). Parallel deflation entails the lowest computational cost (just over 200 iterations) but shows a performance near classical deflation with inverse ordering, marginally improving on the MMSE's average SER at high SNR.

7 CONCLUSIONS

We have exploited the discrete nature of digital communication signals to address the deflation-based blind source extraction in underdetermined mixtures. As already noticed in other works (e.g., [1]), some type of non-linear processing is necessary to extract all sources satisfactorily. Herein, projection on the signal alphabet before deflation has been shown to ameliorate the performance of linear extraction, with an alphabet-based criterion (CP) clearly outperforming the traditional CM principle, even for sources verifying the CM assumption. An alphabet-matched linear extraction criterion followed by projection on the signal alphabet can considerably improve the performance of classical regression-based deflation in extracting all sources from an underdetermined mixture with a reasonably low probability of error. Also, the gradual interference suppression of classical deflation seems to have

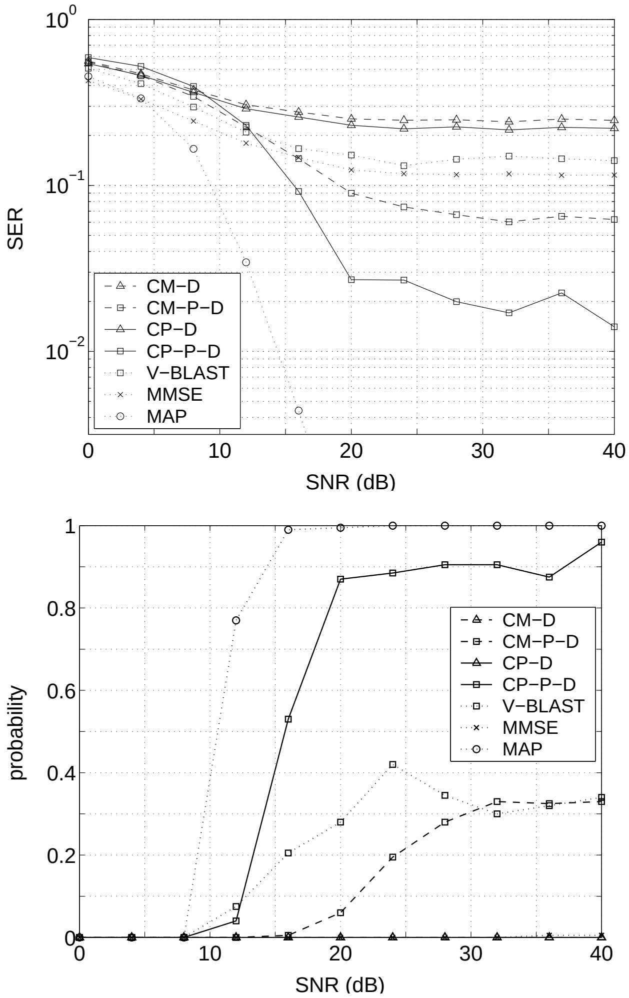

Figure 1: Source extraction results in the underdetermined (3×4) scenario with QPSK sources, signal blocks of 150 symbols and 200 Monte Carlo runs. Top: average separator output SER. Bottom: probability of extracting the 4 sources with SER < 0.1.

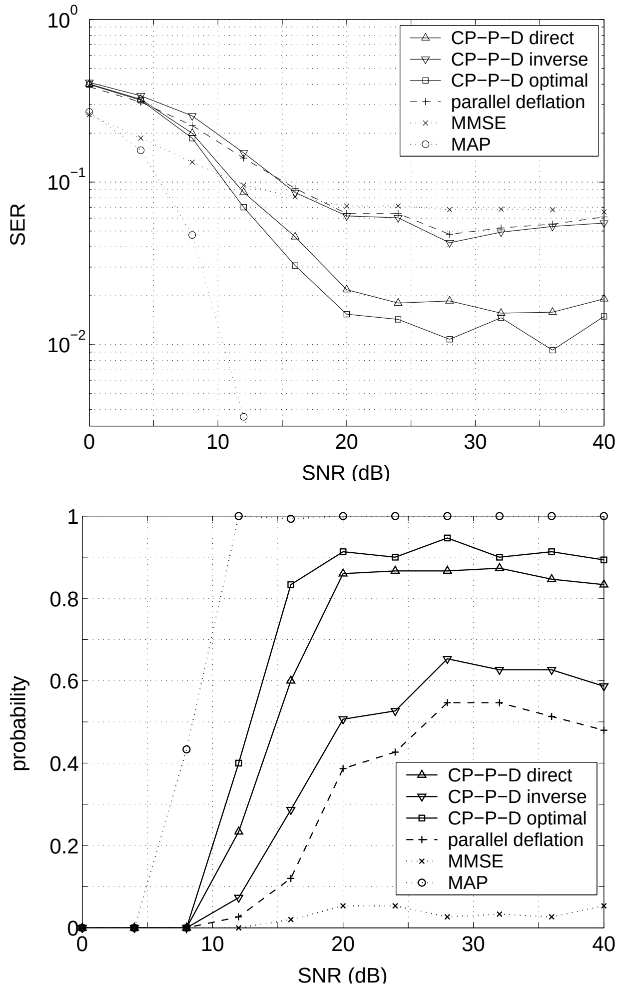

Figure 2: Source extraction results in the underdetermined (4×6) scenario with three BPSK and three 3-PSK sources, signal blocks of 150 symbols and 150 Monte Carlo runs. Top: average separator output SER. Bottom: probability of extracting the 6 sources with SER < 0.1.

a more significant positive impact than the reduced error accumulation of parallel deflation. The further performance enhancement provided by the proposed method for optimising the extraction order may not compensate for the additional computational cost. The method is reminiscent of the V-BLAST technique [5], but requires no training and can handle scenarios of less sensors that sources with possibly different modulations.

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