Exploiting non-Gaussianity in blind identification and equalisation of MIMO FIR channels

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Abstract: The problem of blind identification and equalisation (BIE) of finite impulse response (FIR) channels in multiuser digital communications is investigated. The non-Gaussian nature and statistical independence of the users' data streams is exploited by resorting to blind signal separation (BSS) based on higher-order statistics (HOS). Two such techniques are put forward. The first technique is composed of an extension to the multiuser case of a second-order BIE method, followed by a BSS-based space-equalisation step. The second technique achieves joint space–time equalisation through the direct application of a HOS-based BSS method followed by a blind identification algorithm. In a number of numerical experiments, the first procedure proves less costly and more effective for short data records. Despite their computational complexity, interesting features such as constellation-independent channel identification and symbol recovery, and robustness to ill-conditioned channels in high SNR environments render HOS-BSS based BIE methods an effective alternative to BIE techniques exploiting other spatio-temporal structures.

1 Introduction

In digital communications, linear distortion effects such as multipath propagation and limited bandwidth cause intersymbol interference (ISI) in the received signal, producing errors in symbol detection. A variety of equaliser designs can be employed to compensate for the channel effects [1]. As opposed to traditional techniques, blind channel identification and equalisation (BIE) methods do not require training sequences, and are thus able to use the bandwidth resources more efficiently and to perform in a wider range of communication environments. Due to their many desirable properties [2], blind methods have aroused great research interest.

Tong *et al*. first proved [3] that non-minimum phase (NMP) finite-impulse response (FIR) channels can be identified using only second-order statistics (SOS) if the received signal exhibits cyclostationarity. Cyclostationarity naturally leads to the so-called single-input multiple-output (SIMO) model, a multichannel signal structure with one input (the transmitted symbol sequence) and several outputs. By relying only on the subspace information contained within the sensor second-order correlation matrix, BIE is possible in SIMO systems [3, 4].

In multiuser communication environments (e.g. cellular wireless systems) the co-channel interference (CCI) caused by other users simultaneously transmitting across the same medium adds to multipath-induced ISI. To ensure reliable detection, space–time equalisation must be performed. Time equalisation aims at ISI removal, whereas space equalisation involves CCI elimination and the extraction of the signal(s) of interest. The exploitation of temporal and/or spatial diversity (fractional sampling and/or multiple sensors) results in multiple-input multiple-output (MIMO) signal models. Direct extensions of subspace-based SIMO methods to the MIMO case achieve time equalisation but are generally unable to separate the different source data streams, i.e. CCI remains in the form of an instantaneous linear mixture of the transmitted symbols [2, 5, 6]. To separate this spatial mixture, the fact that digital communication signals possess a finite alphabet (FA) can be exploited [2]. In a direct-sequence code-division multiple access (DS/CDMA) system, [6] reports an unsatisfactory performance of one such FA-based method, with probability of error well above 10% even in the noise-free case. Nevertheless, the spatial mixture can be resolved with the aid of the users' signature sequences [6], which are typically known in a CDMA scenario. This semi-blind method is not applicable to a general (i.e. using a multiple-access technique other than DS/CDMA) multiuser digital communication environment. The method of [6] is blind in that it spares training sequences. However, the use of signature sequences leads to a particular factorisation of the channel matrix, whereas fully blind methods generally avoid such parameterisations. Precisely there lies the robustness of these methods to deviations from the assumed prior information (e.g. calibration errors in beamforming) [7].

A more generic, fully blind approach sparing the prior knowledge of the users' signature sequences or FAs follows from the plausible hypothesis that the signals transmitted by different users are statistically independent. Hence, the remaining spatial mixture after the SOS-MIMO stage adopts a model of blind source separation (BSS) of instantaneous linear mixtures. In addition, digital communication signals are non-Gaussian, typically showing sub-Gaussian (or platykurtic [8]) probability density functions (pdfs), so BSS methods based on higher-order statistics (HOS) are applicable. In the case where the transmitted symbols are independent and identically distributed (i.i.d.), the source extraction can directly be solved by HOS-based BSS

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techniques, as shown in [9–11]. However, the methods presented therein are not designed to identify the channel. A channel estimate may prove useful in a variety of tasks, such as power control, source localisation, propagation characterisation, or as a sensible initialisation for an adaptive receiver.

This contribution discusses the exploitation of the non-Gaussian i.i.d. source property in the FIR–MIMO BIE problem. In particular, we study two techniques which rely on such an assumption through the application of BSS. The first technique is composed of the extension to the MIMO case of a SOS-based SIMO method, completed by a BSS-based space-equalisation stage. The second technique consists of joint space–time equalisation through the direct application HOS-based BSS followed by a suitable algorithm for channel identification. The benefits and drawbacks of exploiting non-Gaussianity are also highlighted throughout.

A signal model is presented which will be used in the mathematical developments. The two BIE methods are put forward and simulation results are reported. Other relevant issues are also discussed.

Notations

$\mathbb{C}$ is the set of complex numbers. Vectors and matrices are represented, respectively, by boldface lower-case and upper-case symbols. $(\boldsymbol{A})_{ij}$ denotes the (i, j)-element of matrix $\boldsymbol{A}$. Symbol $\boldsymbol{I}_n$ refers to the $n \times n$ identity matrix, and

$$\boldsymbol{e}_i^{(n)} = [\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{n-i}]^{\mathrm{T}}$$

is the ith canonic basis vector of $\mathbb{C}^n$. Superindices $(\cdot)^*$, $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{-1}$ and $(\cdot)^{\dagger}$ indicate the complex conjugate, transpose, Hermitian (conjugate-transpose), inverse and Moore–Penrose pseudoinverse operators, respectively. E[·] stands for mathematical expectation, and $\otimes$ denotes the Kronecker product.

2 Signal model

The signal model of [4] is extended to the multiple-input case. An oversampled single-sensor receiver is considered, although the model also holds for spatially separated multiple physical sensors. The system assumptions are:

(i) K data sources simultaneously transmit mutually-independent information-bearing non-Gaussian i.i.d. symbols $\{s_{k,m}\}_{k=1}^{K} \in \mathbb{C}$ at a known rate $1/T$ bauds, with $\mathrm{E}[s_{k,\cdot}] = 0$ and $\mathrm{E}[|s_{k,\cdot}|^2] = 1$.
(ii) The impulse responses $h_k(t)$ representing the propagation between the kth source and the sensor (including the effects of the transmitter and receiver filters, carrier-pulse shaping, etc.) span at most $M + 1$ data symbols.
(iii) The additive measurement noise $v(t)$ is white, zero-mean and uncorrelated with the data sequences; its variance is σ^2.

The implicit source power normalisation in assumption (i) stems from the fact that a complex scalar can be interchanged between the channel and the data without altering the received signal. This scalar factor is an admissible indeterminacy in blind equalisation, and cannot be resolved without resorting to further prior information.

In contrast to [2], herein source alphabets can be assumed unknown and not necessarily identical for all users; neither the alphabets need be constant modulus. The source data need not even be discrete. We only require that their kurtosis [8] be different from zero (at most one non-kurtic source is allowed [12]). Assumption (ii) demands channel stationarity, at least over the observation window. This hypothesis is verified in time non-selective scenarios, such as block-fading multipath channels, whose coherence time is large compared to the baud period [13].

With the above assumptions, the continuous-time complex baseband received signal can be expressed as

$$x(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} s_{k,m} h_k(t - mT) + v(t) \tag{1}$$

Sampling at a rate $1/T_s = L/T$, with L integer, from an initial instant $t_0 = 0$ s (without loss of generality) yields

$$x_n^{(i)} = \sum_{k=1}^{K} \sum_{m=0}^{M} s_{k,n-m} h_{k,m}^{(i)} + v_n^{(i)}, \quad i = 0, \ldots, L-1 \tag{2}$$

in which $x_n^{(i)} = x(iT_s + nT)$, $h_{k,n}^{(i)} = h_k(iT_s + nT)$ and $v_n^{(i)} = v(iT_s + nT)$. Hence, fractionally-spaced sampling effectively generates L virtual channels excited by the same input. Let us now store N consecutive output samples of virtual channel i in vector $\boldsymbol{x}_n^{(i)} = [x_n^{(i)}, \ldots, x_{n-N+1}^{(i)}]^{\mathrm{T}}$. Parameter N is referred to as the smoothing factor [14] or stacking level [9]. Similarly, gather the N samples of the L virtual channel outputs in vector $\boldsymbol{x}_n = [\boldsymbol{x}_n^{(0)^{\mathrm{T}}}, \ldots, \boldsymbol{x}_n^{(L-1)^{\mathrm{T}}}]^{\mathrm{T}}$ (with similar notations for the noise vector $\boldsymbol{v}_n$). Then, the following matrix model holds:

$$\boldsymbol{x}_n = \boldsymbol{H}\boldsymbol{s}_n + \boldsymbol{v}_n \tag{3}$$

where $\boldsymbol{s}_n = [\boldsymbol{s}_{1,n}^{\mathrm{T}}, \ldots, \boldsymbol{s}_{K,n}^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{s}_{k,n} = [s_{k,n}, \ldots, s_{k,n-N-M+1}]^{\mathrm{T}}$; $\boldsymbol{H} = [\boldsymbol{H}_1, \ldots, \boldsymbol{H}_K]$ is the $LN \times K(M+N)$ channel filtering matrix, with $\boldsymbol{H}_k = [\boldsymbol{H}_k^{(0)^{\mathrm{T}}}, \ldots, \boldsymbol{H}_k^{(L-1)^{\mathrm{T}}}]^{\mathrm{T}}$, $\boldsymbol{H}_k^{(i)}$ representing the $N \times (M+N)$ Toeplitz convolution matrix associated with the linear filter $\boldsymbol{h}_k^{(i)} = [h_{k,0}^{(i)}, \ldots, h_{k,M}^{(i)}]^{\mathrm{T}}$. To abbreviate, in the sequel we denote $P \triangleq LN$, $C \triangleq M+N$ and $D \triangleq K(M+N) = KC$.

The objective of BIE is to estimate $\boldsymbol{H}$ (blind channel identification) and $\boldsymbol{s}_n$ (blind channel equalisation [ISI cancellation] and source separation [CCI cancellation]) from the only observation of the received vector $\boldsymbol{x}_n$. These tasks are equivalent to recovering the channel coefficient vector $\boldsymbol{h} = [\boldsymbol{h}_1^{\mathrm{T}}, \ldots, \boldsymbol{h}_K^{\mathrm{T}}]^{\mathrm{T}}$, with $\boldsymbol{h}_k = [\boldsymbol{h}_k^{(0)^{\mathrm{T}}}, \ldots, \boldsymbol{h}_k^{(L-1)^{\mathrm{T}}}]^{\mathrm{T}}$, and the source vector

$$\boldsymbol{s} = E_1^{\mathrm{H}} \boldsymbol{s}_n = [s_{1,n}, \ldots, s_{K,n}]^{\mathrm{T}} \tag{4}$$

where $E_i = \boldsymbol{I}_K \otimes \boldsymbol{e}_i^{(C)}$.

A necessary condition for blind identifiability is that the filtering matrix be full column rank, which can occur only if $\boldsymbol{H}$ has more rows than columns, $P \geq D$, or, equivalently, $L > K$ and $N \geq KM/(L-K)$. This condition is not sufficient. It is required that polynomial matrix $\boldsymbol{H}(z)$ be 'irreducible and column reduced', where $(\boldsymbol{H}(z))_{ij}$ is the z-transform of $h_j^{(i)}$ [5].

Note that with the information of assumptions (i)–(iii) we can obtain at best a channel estimate $\hat{\boldsymbol{H}}$ such that $\hat{\boldsymbol{H}}^{\dagger}\boldsymbol{H} = \boldsymbol{\Gamma}_K \otimes \boldsymbol{I}_C$, where $\boldsymbol{\Gamma}_K$ is an arbitrary $K \times K$ permutation matrix with unit-norm nonzero entries; signal blocks of different users present an order indeterminacy, which can only be surmounted if further information is available (e.g. users' signature sequences in a CDMA system).

3 SOS-based time equalisation and BSS-based space equalisation

3.1 Multiuser extension of SIMO methods

Tong *et al.* [3] realised that blind channel identification of NMP FIR channels is possible from SOS alone in the single-user cyclostationary case, which results in the SIMO signal model. (The signal model in [3] is slightly different from that presented in the previous Section (with $K = 1$). Tong's signal model involves a different arrangement for the signal vectors and channel matrix, and allows for noninteger (fractional) values for the stacking level. However, both models are totally analogous, so that we can use the model of Section 2 without loss of generality.) Their approach takes advantage of the particular structure of the observed-vector correlation matrix $\boldsymbol{R}_x(m) = \mathrm{E}[\boldsymbol{x}_n \boldsymbol{x}_{n-m}^{\mathrm{H}}]$ at two different lags $(m = 0, 1)$. The direct application of this blind identification method to the MIMO case yields the following identifiability result [15].

Theorem 1: Suppose that $\boldsymbol{H}$ and $\boldsymbol{s}_n$ satisfy the linear model (3) and its constraints (i)–(iii). Then $\boldsymbol{H}$ is determined from $\boldsymbol{R}_x(0)$ and $\boldsymbol{R}_x(1)$ up to a post-multiplicative factor of the form $\boldsymbol{Q} \otimes \boldsymbol{I}_C$, where $\boldsymbol{Q} \in \mathbb{C}^{K \times K}$ is a $K \times K$ unitary matrix.

A similar indeterminacy is observed in the multiuser extension [2, 16] of the subspace method of [4], in which $\boldsymbol{Q}$ becomes an arbitrary $K \times K$ invertible matrix. Indeed, theorem 1 may be generalised to the MIMO extension of any SIMO BIE method [2].

According to the above result, the channel estimated by the extended Tong's method is of the form $\tilde{\boldsymbol{H}} = \boldsymbol{H}(\boldsymbol{Q} \otimes \boldsymbol{I}_C)$, with $\boldsymbol{Q}$ an unknown $K \times K$ unitary matrix. Should we want to carry out soft-symbol detection at this stage, the resulting zero-forcing (ZF) equaliser output would be

$$\boldsymbol{y}_n = \tilde{\boldsymbol{H}}^{\dagger} \boldsymbol{x}_n = (\boldsymbol{Q}^H \otimes \boldsymbol{I}_C)\boldsymbol{s}_n + \tilde{\boldsymbol{v}}_n \tag{5}$$

in which

$$\tilde{\boldsymbol{v}}_n = \tilde{\boldsymbol{H}}^{\dagger} \boldsymbol{v}_n \tag{6}$$

Now, defining $\boldsymbol{y} = E_1^{\mathrm{H}} \boldsymbol{y}_n$, system (5) becomes

$$\boldsymbol{y} = \boldsymbol{Q}^{\mathrm{H}} \boldsymbol{s} + \tilde{\boldsymbol{v}} \tag{7}$$

where $\tilde{\boldsymbol{v}} = E_1^{\mathrm{H}} \tilde{\boldsymbol{v}}_n$, and $\boldsymbol{s}$ is given by (4).

3.2 BSS-based space equalisation

Equation (7) represents a noisy unitary instantaneous linear mixture of the source symbols. That is, CCI elimination requires further processing. Since the components of $\boldsymbol{s}$ are statistically independent [assumption (i)], (7) corresponds to a BSS problem of instantaneous linear mixtures [17, 18]. Due to the i.i.d. assumption, SOS-based BSS methods fail, but the source non-Gaussianity can still be exploited through HOS. A few remarks indicate that HOS-based BSS seems well suited as a second processing step:

3.2.1 Complexity reduction: The BSS problem at this second stage has size $K \times K$, which is considerably reduced compared with the original dimensions of the BIE system (3).

3.2.2 Robustness to ill-conditioned channels: In the single-user case, the so-called uniform performance property enjoyed by many BSS methods [19] translates into a robust performance for ill-conditioned channels [9]. Note, however, that uniform performance is only expected to hold in the noiseless case [19].

3.2.3 Noise 'Gaussianisation': The central limit theorem and (6) guarantee that the equalised noise $\tilde{\boldsymbol{v}}$ will be close to Gaussian, even if the actual sensor noise $\boldsymbol{v}_n$ is not. The well known HOS immunity to Gaussian noise would then result in an increased robustness of the BIE method not only to Gaussian noise but also to other kinds of non-Gaussian noise, such as impulsive interference.

In the simulations of Section 5, we employ the joint approximate diagonalisation of eigenmatrices (JADE) BSS method [7]. This choice is somewhat arbitrary; we are concerned with the application of BSS as a general strategy, rather than assessing which particular BSS method provides the best performance. JADE optimises a HOS cost function through the joint diagonalisation of a particular set of fourth-order cumulant tensor 'slices'.

Once $\boldsymbol{Q}$ has been obtained via a HOS–BSS method, the full channel estimate can be calculated as $\hat{\boldsymbol{H}} = \tilde{\boldsymbol{H}}(\boldsymbol{Q}^{\mathrm{H}} \otimes \boldsymbol{I}_C)$. From the channel estimate, soft-symbol detection can then be accomplished from (3) as $\hat{\boldsymbol{s}}_n = \boldsymbol{G}^{\mathrm{H}} \boldsymbol{x}_n$ with the ZF and minimum mean square error (MMSE) equalisers

$$\boldsymbol{G}_{\mathrm{ZF}} = (\hat{\boldsymbol{H}}\hat{\boldsymbol{H}}^{\mathrm{H}})^{-1}\hat{\boldsymbol{H}} \tag{8}$$

$$\boldsymbol{G}_{\mathrm{MMSE}} = \boldsymbol{R}_x(0)^{-1}\hat{\boldsymbol{H}} \tag{9}$$

whose subspace version from the channel matrix singular value decomposition is given in [6].

Steps 2–5 of Table 1 summarise the BSS and detection stages, which, in combination with the extended Tong method, complete the first FIR–MIMO BIE algorithm proposed in this paper.

4 BSS-based joint space–time equalisation and channel identification

The i.i.d. assumption in (i) makes the components of the source vector $\boldsymbol{s}_n$ in (3) statistically independent. From this perspective, (3) itself can also be considered as a BSS model of instantaneous linear mixtures, and thus BSS techniques may be directly applied to resolve it [11].

The whitening step of Tong's method provides the outputs [15]

$$\boldsymbol{z}_n = \boldsymbol{W}\boldsymbol{x}_n = \boldsymbol{V}\boldsymbol{s}_n + \boldsymbol{w}_n \tag{10}$$

where $\boldsymbol{w}_n = \boldsymbol{W}\boldsymbol{v}_n$ and $\boldsymbol{W}$ represents the whitening matrix. In a second step, a HOS-based BSS method, such as JADE [7], can estimate the unitary mixing matrix $\boldsymbol{V}$. Detection can be carried out through ZF/MMSE equalisers (8)/(9).

Since in this case the BSS method operates over all the D whitened components, the complexity reduction remarked in the previous Section is lost. However, the key point to note in the direct application of BSS techniques to the BIE model is related to the source scale and order indeterminacies inherent to the BSS problem [17, 18]. These indeterminacies mean that a blind separation method can provide any solution $\hat{\boldsymbol{V}}$ such that $\hat{\boldsymbol{V}}^{\dagger}\boldsymbol{V}$ is an arbitrary permutation matrix with unit-norm non-zero entries. In our particular model (3), the arrangement and scale of the recovered source components as well as the corresponding columns of the filtering matrix are crucial, especially for

Table 1: Algorithm for SOS-based time equalisation and BSS-based space equalisation

1. Obtain first estimate of the filtering matrix $\tilde{\boldsymbol{H}}$ from the extended Tong method [15].
2. Compute ISI-free output (5).
3. Estimate matrix $\boldsymbol{Q}$ from (7) with a HOS-based BSS method.
4. Update estimate of channel matrix as $\hat{\boldsymbol{H}} = \tilde{\boldsymbol{H}}(\boldsymbol{Q}^{\mathrm{H}} \otimes \boldsymbol{I}_C)$.
5. Detect CCI-free source symbols [(8) and (9)].

channel identification purposes. Hence, the solution obtained via BSS needs to be refined if it is to be useful in the BIE scenario.

We propose the algorithm outlined in Table 2. First, the recovered source vector components are normalised to unit variance, and the component with the largest absolute normalised kurtosis is chosen. Given a unidimensional observation $x = s + v$, then

$$\tilde{\kappa}_4^x = \tilde{\kappa}_4^s \left(\frac{\text{SNR}}{1 + \text{SNR}} \right)^2$$

where $\tilde{\kappa}_4^x = \kappa_4^x / (\kappa_2^x)^2$, $\text{SNR} = \kappa_2^s / \kappa_2^v$ and κ_n^x and $\tilde{\kappa}_n^x$ represent the nth-order cumulant and normalised cumulant, respectively, of x [8]. Hence, if all sources have the same distribution, the highest normalised kurtosis criterion selects the least noisy component. Next, the correlation function between that and all other components is computed in turns. If the maximum absolute value of the correlation function is above certain threshold, the two components are considered to belong to the same source, their relative delay and phase being given by the delay and phase of their joint correlation function at its peak. (In the simulations of Section 5, an initial threshold value of 0.7 was used, with a multiplicative reduction factor (step 5 in Table 2) of 0.95.) Note that since the relative delays between two components of the same user's signal lie in the interval $[-C+1, C-1]$, the cross-correlation functions only need to be computed between those lag limits, with the consequent reduction in complexity. The components of the source vector and the channel matrix are then scaled and ordered accordingly. The process is repeated until no more source components remain to be arranged. In an ideal situation (perfect estimation), this algorithm outputs a channel estimate $\hat{\boldsymbol{H}}$ such that $\hat{\boldsymbol{H}}^{\dagger}\boldsymbol{H} = \boldsymbol{\Gamma}_K \otimes \boldsymbol{I}_C$.

Table 2: Algorithm for BSS-based joint space–time equalisation and channel identification

Repeat steps below until no more estimated source components remain to be ordered:

1. Select among remaining sources the component with largest normalised kurtosis, $\hat{s}_i$.
2. Estimate (e.g. via time averaging) the cross-correlation functions

$$\boldsymbol{R}_{\hat{s}_i \hat{s}_j}(m) = \mathrm{E}[\hat{s}_i(n)\hat{s}_j^*(n-m)]$$

for j within the group of components still to be arranged.

3. Obtain the lag position m_{ij} of the largest absolute value $|\rho_{ij}|$ of $\boldsymbol{R}_{\hat{s}_i \hat{s}_j}$

$$m_{ij} = \arg\max_m |\boldsymbol{R}_{\hat{s}_i \hat{s}_j}(m)|$$

$$\rho_{ij} = \boldsymbol{R}_{\hat{s}_i \hat{s}_j}(m_{ij})$$

4. If $|\rho_{ij}| >$ threshold, source pair $(\hat{s}_i, \hat{s}_j)$ belongs to the same user.
   (a) Correct phase: multiply $\hat{s}_j$ by $e^{j\angle\rho_{ij}}$; multiply the jth column of $\hat{\boldsymbol{H}}$ by $e^{-j\angle\rho_{ij}}$.
   (b) Rearrange the elements of $\hat{\boldsymbol{s}}$ and the columns of $\hat{\boldsymbol{H}}$ according to the ordering of m_{ij}.
5. If no source pair was detected, reduce the threshold and return to step 4.

The above algorithm is based on the equalisation methods 'A' and 'C' of ([11], Section 5), but it improves them in that it is also able to accomplish channel identification.

5 Simulation results

A few numerical experiments illustrate the behaviour of the MIMO BIE methods presented in the previous Sections. We first define a number of performance parameters. A natural choice for the signal-to-noise ratio (SNR) is

$$\text{SNR} = \frac{\text{trace}(\boldsymbol{H}\boldsymbol{R}_s(0)\boldsymbol{H}^{\text{H}})}{\text{trace}(\boldsymbol{R}_v(0))} = \frac{1}{\sigma^2 P}\text{trace}(\boldsymbol{H}\boldsymbol{H}^{\text{H}}) \qquad (11)$$

which corresponds to the average source power contribution over the average noise power in the received signal. To measure the quality of the channel identification and the space–time equalisation results, we choose the channel mean square error (CMSE) and the average signal mean square error (SMSE), respectively, which are defined as

$$CMSE = \frac{\|\hat{\boldsymbol{h}} - \boldsymbol{h}\|^2}{\|\boldsymbol{h}\|^2} \qquad (12)$$

$$SMSE = \frac{1}{D}\mathrm{E}\left[\|\hat{\boldsymbol{s}}_n - \boldsymbol{s}_n\|^2\right] \qquad (13)$$

The symbol error rate (SER) is computed as the number of erroneous symbols in the components of $\hat{\boldsymbol{s}}_n$ over the total number of symbols in $\boldsymbol{s}_n$. Before calculating these performance parameters, the estimated channel matrix and source vector are first rearranged to 'match' the original channel matrix and source vector, in a bid to correct the $\boldsymbol{\Gamma}_K \otimes \boldsymbol{I}_C$ ambiguity term. It is important to note that this rearrangement is based purely on the comparison of consecutive C-column blocks of the estimated and original channel matrices, so that the BIE results cannot possibly be altered (improved) in this process.

5.1 Performance against sample size

The first simulation tests the extension of Tong's method followed by BSS on (7) (ETBSS), and the full BSS method on (3) with the blind identification algorithm of Section 4 (FBSS). Two 4-QAM signals are transmitted over a dispersive multipath channel with a short delay spread of $M = 2$ symbol periods. Reception takes place in additive complex Gaussian noise. N_d symbol periods are observed, with oversampling factor $L = 6$ and stacking level $N = 2$. The channel coefficients are drawn from a complex Gaussian distribution, forming a fixed 12×8 channel matrix with condition number $\text{cond}(\boldsymbol{H}) = 5$. Performance parameters are averaged over ν Monte Carlo (MC) runs, with independent source and noise realisations at each run, and maintaining $\nu N_d = 10^4$. Figure 1 shows the CMSE and SMSE results for a varying observation window length N_d and several SNRs. At high enough SNR, ETBSS shows good performance for a low sample size. FBSS needs around 300 samples to provide satisfactory results, and then consistently outperforms the other method, becoming about twice as efficient. Figure 1*b* also shows that the methods tend asymptotically to the large-sample MMSE at each SNR value. In this experiment, SER counts are zero for both methods from $\text{SNR} = 30\,\text{dB}$ and $N_d > 300$ observed symbol periods, approximately.

5.2 Performance against noise level

The environment of the second simulation tests the effects of varying noise levels for different sample lengths, with

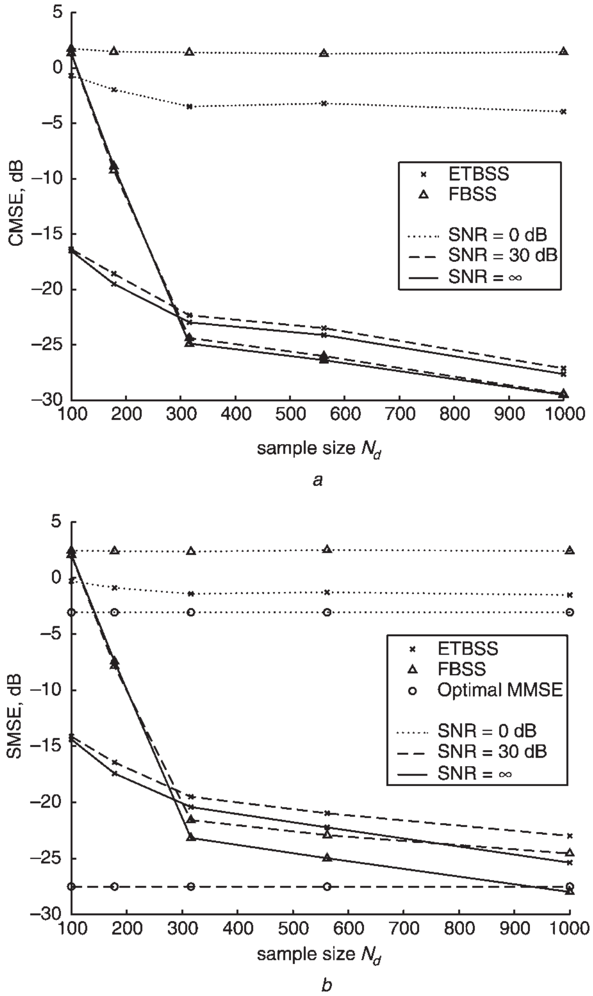

Fig. 1 *Performance against sample size for two 4-QAM sources, additive Gaussian noise, $M = 2$, $L = 6$, $N = 2$, $cond(\boldsymbol{H}) = 5$, ν MC runs, $\nu N_d = 10^4$*

a CMSE
b SMSE with MMSE detection

$\nu N_d = 10^5$. Three 16-QAM modulations propagate in a more severe frequency-selective channel of order $M = 5$. We choose $L = 12$ and $N = 2$, which result in a 24×21 channel matrix with $\text{cond}(\boldsymbol{H}) = 30$. Figure 2 shows that the ETBSS begins to obtain satisfactory BIE results from about SNR = 20 dB, even for low sample size, whereas FBSS requires a few thousand samples to start performing. However, for long observation windows, FBSS tolerates a noise level of around 10 dB higher than ETBSS. Both methods approach the optimal large-sample MMSE asymptotically, as displayed in Figs. 2*b* and 2*c*. In the SER plots, the 'optimal MMSE' curve corresponds to the probability of symbol error in the optimum detection (for an AWGN channel [13]) of a single component with MSE equal to the large-sample MMSE in the given simulation conditions (channel matrix and SNR).

5.3 Performance against noise distribution

Figure 3 explores the impact of the noise distribution on the BIE results, for large sample size ($N_d = 10^4$) at various SNRs. The sensor output is corrupted by additive noise with complex generalised Gaussian distribution (CGGD) of parameter α, whose pdf is given by $p(z) \propto \exp(-|z|^{\alpha})$. The CGGD becomes the complex Gaussian distribution for $\alpha = 2$, a super-Gaussian distribution for $\alpha < 2$ (e.g. the complex Laplacian variable for $\alpha = 1$), and a sub-Gaussian distribution for $\alpha > 2$. The methods' BIE results are

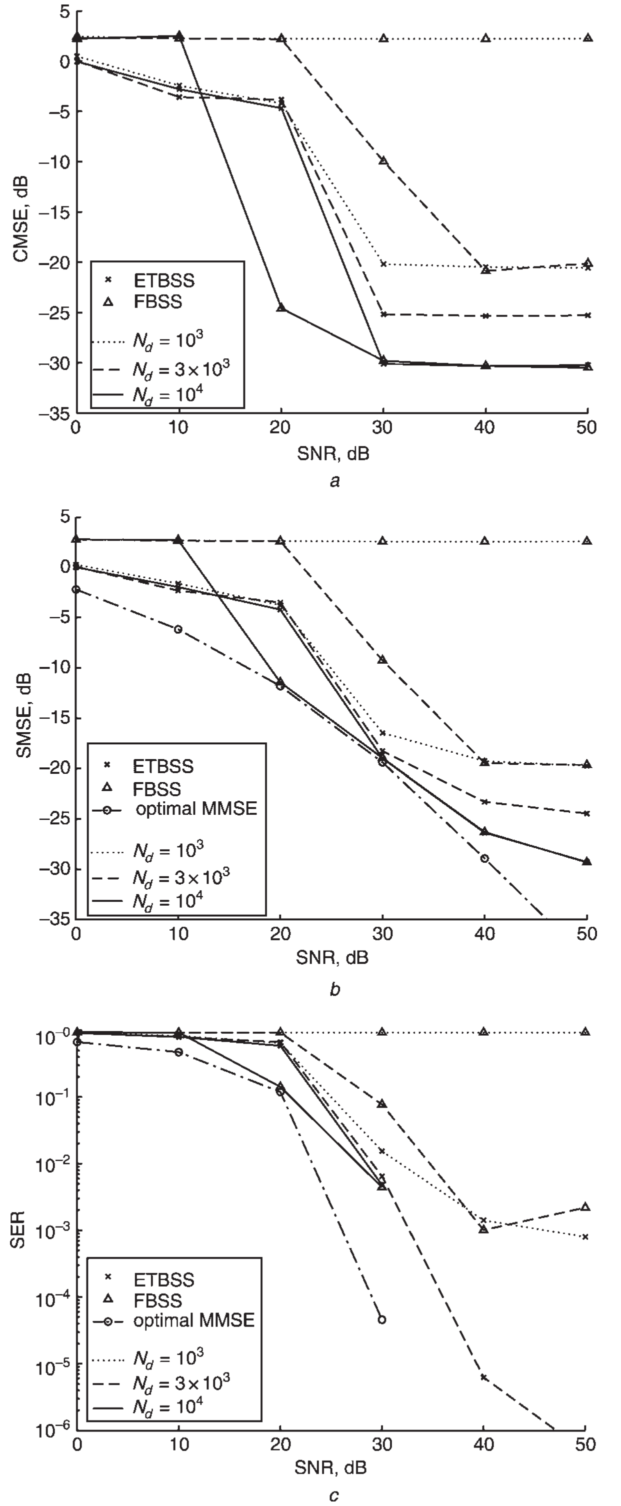

Fig. 2 *Performance against SNR for three 16-QAM sources, additive Gaussian noise, $M = 5$, $L = 12$, $N = 2$, $cond(\boldsymbol{H}) = 30$, ν MC runs, $\nu N_d = 10^5$*

a CMSE
b SMSE with MMSE detection
c SER with MMSE detection

virtually identical over the tested range of noise distributions.

5.4 Performance against channel conditioning

The effects of the channel matrix conditioning are assessed in a final experiment, whose outcome is shown in Fig. 4. At each MC iteration, a channel matrix of a given condition

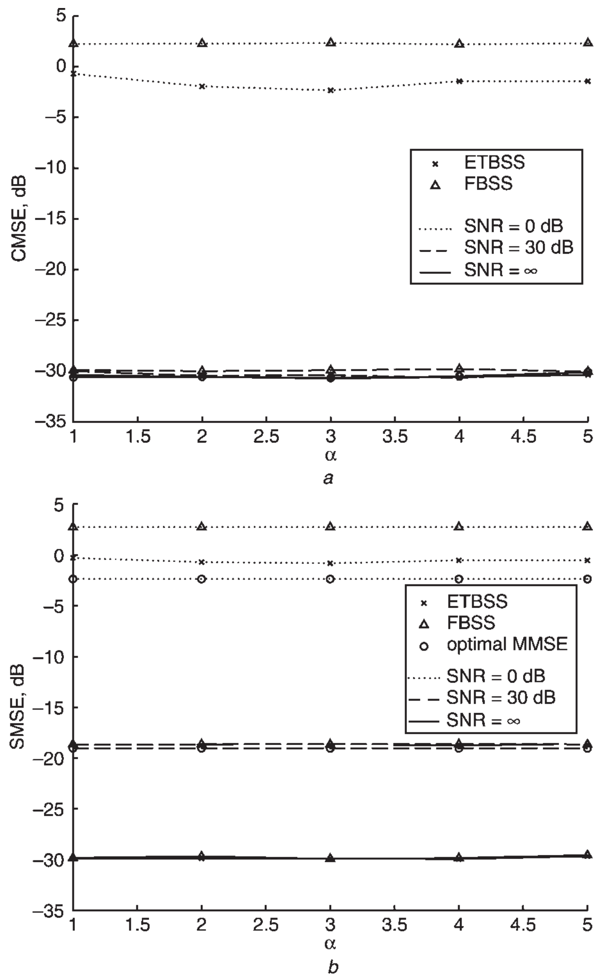

Fig. 3 *Performance against noise distribution for additive noise with CGGD of parameter α, three 16-QAM sources, $M = 5$, $L = 12$, $N = 2$, cond($\boldsymbol{H}$) = 30, $N_d = 10^4$ 10 MC runs*

a CMSE

b SMSE with MMSE detection

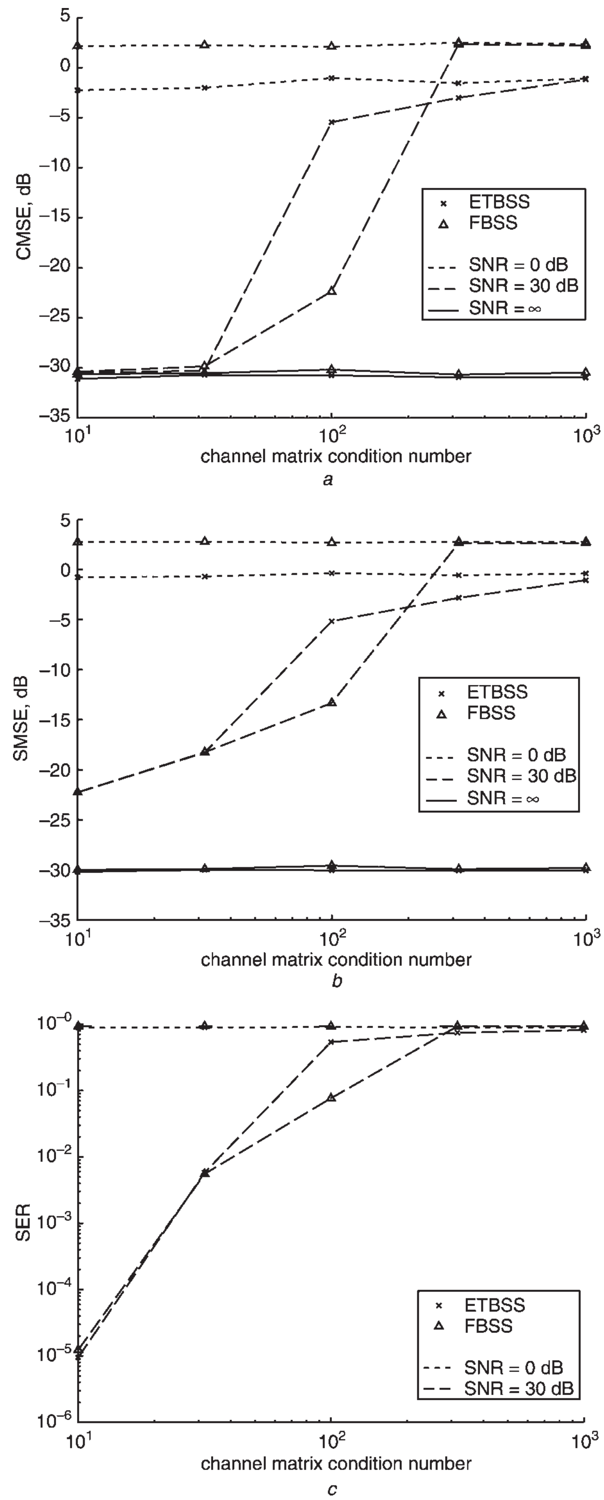

Fig. 4 *Performance against channel conditioning, for three 16-QAM sources, additive Gaussian noise, $M = 5$, $L = 12$, $N = 2$, $N_d = 10^4$, 10 MC runs*

a CMSE

b SMSE with MMSE detection

c SER with MMSE detection

number as well as independent source and noise realisations are randomly generated. For finite SNR, performance worsens as cond($\boldsymbol{H}$) increases. The ill conditioning of the channel matrix amplifies the noise in the whitening process, hampering the HOS processing stage, which 'sees' a lower SNR. In the noiseless case, no variation with the channel matrix condition number is observed. For illustration and comparison, the characteristics of some channels used in this paper and elsewhere in the literature are summarised in Table 3.

6 Discussion

A number of issues deserve special treatment, and are discussed next.

6.1 Computational complexity and choice of BSS method

The high cost involved in the computation of the higher-order cumulants/moments is probably the weakest aspect of HOS-based techniques. After Tong's method, JADE requires the calculation of the K^4 elements of the fourth-order cumulant tensor, followed by the diagonalisation of a $K^2 \times K^2$ matrix made from such cumulants. Consequently, the direct application of BSS exhibits a C^4-fold increase in computations. Indeed, JADE becomes computationally prohibitive for source vectors with many components, which may easily arise in more realistic scenarios with large delay spreads. Less costly schemes such as the FastICA algorithm [18] may prove more convenient in these practical situations. The methods of [12] and [20], which can be used in real-valued mixtures, show a complexity of the order of $K^{5/2}$ flops per vector sample. For coloured sources with different spectral content, computationally efficient BSS techniques using only SOS [17, 18, 21] are feasible after the application of a subspace method not relying on the i.i.d. assumption (e.g. [4]).

Table 3: Some channels used in the literature and in this paper

Reference	(K, L, M, N)	Description	Size	cond(**H**)
[3]	(1, 4, 5, 5)	SIMO, raised cosine carrier pulses, three-ray multipath, NMP subchannels	20×10	55.86
[9], case 1	(1, 2, 11, 11)	SIMO, similar impulse response to [3], NMP subchannels	22×22	2.96×10^4
[9], case 2	(1, 4, 5, 5)	SIMO, NMP subchannels	20×10	77.79
[11], example 1	(2, 8, 3, 1)	MIMO, squared-half-cosine carrier pulses, flat fading, NMP subchannels	8×8	181.14
Section 5.1	(2, 6, 2, 2)	MIMO, complex Gaussian channel taps, NMP subchannels	12×8	5
Sections 5.2 and 5.3	(3, 12, 5, 2)	MIMO, complex Gaussian channel taps, NMP subchannels	24×21	30

6.2 Blind identification from channel matrix structure

The blind identification algorithm from the BSS results proposed herein relies on preserving the source vector structure only. The joint exploitation of the block-Toeplitz structure of the channel matrix could lead to a reduction in the sample size required for satisfactory identification results. The minimum required sample size is ultimately limited by the use of HOS.

6.3 Channels with different delay spreads

In realistic communication environments, channel delay spreads of different users are likely to differ. The application of a SOS subspace method would then result, even under perfectly known channel orders, in a BSS problem of convolutive mixtures [5], which is a challenging area currently drawing intense research attention [18].

7 Conclusions

The present work has addressed the BIE of FIR channels in multiuser digital communication systems. The non-Gaussian property and statistical independence of the source data have been successfully exploited through HOS-based BSS techniques for CCI cancellation and for joint ISI-CCI suppression. The two proposed BSS-based techniques exhibit the same asymptotic performance, but the former is computationally more efficient, and proves more effective in short observation windows. Both approaches have shown their robustness, relative to the Gaussian-noise case, against non-Gaussian additive noise and impulsive interference. Other salient features of the BSS approach are its robustness to the channel matrix condition number in high SNR situations and its constellation-independent BIE capabilities. In conclusion, the BSS approach appears to be a strong alternative to FIR-MIMO BIE techniques relying on the exploitation of other spatio-temporal properties such as the users' finite alphabets, constant modulus or signature sequences.

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