IMPROVING MIMO CHANNEL EQUALIZATION WITH INDEPENDENT COMPONENT ANALYSIS

Vicente Zarzoso and Asoke K. Nandi*

Department of Electrical Engineering and Electronics, The University of Liverpool
Brownlow Hill, Liverpool L69 3GJ, UK
{vicente, aknandi}@liv.ac.uk

ABSTRACT

The blind equalization of multi-input multi-output (MIMO) digital communication channels is addressed in this paper. A variety of error sources (e.g., stemming from finite sample size) can cause considerable discrepancies between practical performance and theoretical limits of multichannel linear equalizers. In this contribution, we show that a post-detection strategy based on the statistical tool of independent component analysis (ICA) can notably alleviate these detrimental effects. Thanks to the time redundancies introduced by the wideband multipath channel, ICA-assisted detection can bring MIMO equalization performance closer to the theoretical bounds without the excessive computational complexity of extracting all signal components.

1. INTRODUCTION

Multichannel equalization of digital communication systems has drawn considerable research attention recently. This interest has been spurred by two remarkable discoveries [1–3]: 1) single-input non-minimum phase channels can be blindly identified using only second-order statistics (SOS), and 2) finite-impulse response (FIR) channels accept FIR equalizers. Multichannel structures naturally arise in practical communication systems when exploiting diversity in time (oversampling), space (antenna arrays), and/or code (spreading sequences) [4]. The use of multiple transmit antennas leads to multiple-input multiple-output (MIMO) systems, which offer important performance improvements over conventional transmission schemes. Enhanced spectral efficiency, higher data rates and increased capacity feature among the typical benefits of MIMO systems. To achieve these benefits, MIMO channel impairments such as co-channel interference (CCI) caused by the multiple inputs and intersymbol interference (ISI) due to multipath propagation need to be tackled through suitable equalization techniques. Channel identification and equalization has traditionally been aided by training sequences. However, operating blindly (i.e., without training data) proves beneficial in terms of flexibility and bandwidth resources [5].

Although direct blind equalization is feasible [2, 6–9], a previous channel identification [1,3] may be useful, e.g., for source localization or propagation characterization. The estimated channel can then be used to equalize the received signal. Linear receivers such as the zero forcing (ZF) or the minimum mean squared error (MMSE) detectors offer a reasonable compromise between performance and complexity. In practice, the equalization quality of these linear receivers can notably depart from the theoretical bounds, even when the channel parameters are perfectly known. Sources of error such as the finite-sample estimation of the sensor-output covariance matrix can severely degrade performance. In [10], it is proposed that the statistical tool of independent component analysis (ICA) could be used to ameliorate the impact of channel and covariance matrix estimation errors on equalization quality. A single user of interest was extracted in a short delay-spread (spanning less than half symbol period) multipath CDMA channel. However, this signal model is rather constrained. In general, the simultaneous demodulation of all existing inputs is necessary, particularly if spatial multiplexing is employed at the transmitting end to increase the data throughput. Furthermore, a more realistic characterization of wideband channels should account for longer delay spreads. Long delay spreads make it possible to extract the transmitted data at non-zero equalization delays, which can result in important performance gains relative to zero-delay equalization [6, 8].

This contribution applies ICA to wideband multichannel equalization. We propose the use of ICA for the simultaneous extraction of the input signals at their respective optimal equalization delay. In addition, single-delay extraction renders ICA-assisted equalizers computationally feasible in practice. Illustrative simulations demonstrate the improved tolerance to noise, lower sample size requirements and higher system capacity achieved by ICA-based equalization, which brings the performance of linear receivers closer to their theoretical limits.

2. SIGNAL MODEL

Let us consider a communication system with the following assumptions:

- **(A1)** K co-channel input sources transmit, at a known constant baud rate, zero-mean data symbols $\mathbf{s}(n) = [s_1(n), \ldots, s_K(n)]^{\mathrm{T}} \in \mathbb{C}^K$ with identity covariance matrix;
- **(A2)** a diversity-L receiver with vector output $\mathbf{x}(n) = [x_1(n), \ldots, x_L(n)]^{\mathrm{T}} \in \mathbb{C}^L$;
- **(A3)** FIR channels (including pulse-shaping and receive filter effects) spanning at most $(M+1)$ symbols, with matrix coefficients $\mathbf{H}(k) \in \mathbb{C}^{L\times K}$, $k = 0, 1, \ldots, M$, where the channel order M is assumed to be known and the channel taps fixed over the observation window;
- **(A4)** zero-mean additive noise $\mathbf{v}(n) \in \mathbb{C}^L$ independent of the data sources.

The receiver output components in (A2) are not necessarily associated with spatially-separated receive antennas; 'virtual sensors' can also arise from the oversampling of cyclostationary signals or the use of spreading sequences [3, 4]. This makes the resulting signal model rather general. Assumptions (A3) correspond to block fading channels, typical of scenarios with small to moderate Doppler spread values. According to the above assumptions, the MIMO baseband signal model can be expressed as:

$$\mathbf{x}(n) = \sum_{k=0}^{M} \mathbf{H}(k)\mathbf{s}(n-k) + \mathbf{v}(n). \tag{1}$$

Stacking N consecutive received signal vector samples yields:

$$\mathbf{x}_n = \mathbf{H}\mathbf{s}_n + \mathbf{v}_n \tag{2}$$

with $\mathbf{s}_n = [\mathbf{s}(n)^{\mathrm{T}}, \mathbf{s}(n-1)^{\mathrm{T}}, \ldots, \mathbf{s}(n-M-N+1)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^D$, $\mathbf{x}_n = [\mathbf{x}(n)^{\mathrm{T}}, \mathbf{x}(n-1)^{\mathrm{T}}, \ldots, \mathbf{x}(n-N+1)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{C}^P$, and analogous definition for $\mathbf{v}_n$, with $D \stackrel{\triangle}{=} K(M+N)$ and $P \stackrel{\triangle}{=} LN$. $\mathbf{H} \in \mathbb{C}^{P\times D}$ is the block-Toeplitz matrix associated to the channel taps $\underline{\mathbf{H}} =$

* Supported through a Post-Doctoral Research Fellowship awarded by the Royal Academy of Engineering of the UK.

In *Proc. 6th IMA International Conference on Mathematics in Signal Processing*, Cirencester, UK, Dec. 14-16, 2004, pp. 223-226.

$[\mathbf{H}(0), \mathbf{H}(1), \ldots, \mathbf{H}(M)]$, and musts be full column rank. Obvious necessary conditions for input-signal linear extraction are that $L > K$, which demands sufficient space-time-code diversity, and $N \geqslant KM/(L-K)$, which sets a lower bound on the equalizer length.

The objective of blind MIMO equalization is to estimate the source signals $\mathbf{s}(n)$ from the only observation of the sensor output $\mathbf{x}(n)$.

3. BLIND MIMO CHANNEL IDENTIFICATION

Single-input multiple-output (SIMO) channels can be blindly identified using SOS [1–3]. The extension of SOS-based SIMO blind identification methods to the MIMO case can at most obtain channel estimates of the form $\hat{\mathbf{H}} = \mathbf{H}(\mathbf{I}_C \otimes \mathbf{A}^{-1})$, where $\mathbf{A} \in \mathbb{C}^{K\times K}$ is a unknown invertible matrix, $\otimes$ denotes the Kronecker product, and $\mathbf{I}_C$ is the $(C\times C)$ identity matrix, with $C \stackrel{\triangle}{=} (M+N)$ [5, 11]. In the noiseless case, this channel matrix estimate results in the instantaneous (i.e., ISI-free) linear mixture $\mathbf{y}(n) = \mathbf{A}\mathbf{s}(n)$. This spatial mixture can be inverted, thus completing the MIMO channel estimate, by exploiting the input signals' structural properties such as their finite alphabet, constant modulus, or statistical independence. In the latter case, matrix $\mathbf{A}$ can be identified using ICA [11].

In the sequel, it is assumed that the channel matrix $\mathbf{H}$ (or, equivalently, the channel tap matrix $\underline{\mathbf{H}}$) has been estimated through a suitable blind MIMO identification method (as those of, e.g., [5, 11]). Our primary concern is the estimation of the source signals $\mathbf{s}$ from the sensor output $\mathbf{x}$ by using the identified channel.

4. LINEAR DETECTION

4.1 MMSE equalizer

The maximum likelihood sequence estimator is the optimal detector, but its computational load can be prohibitive in scenarios involving a large number of users or highly dispersive channels [12]. Trading off complexity for performance, linear receivers are based on the estimation of a linear transformation $\mathbf{G} \in \mathbb{C}^{P\times D}$ fulfilling certain (sub)optimality criterion; data are then detected as $\hat{\mathbf{s}}_n = \mathbf{G}^{\mathrm{H}}\mathbf{x}_n$. Since the ZF detector is known to produce undesired noise amplification, we focus our attention on the MMSE equalizer:

$$\mathbf{G}_{\mathrm{MMSE}} = \arg\min_{\mathbf{G}} \mathrm{E}\{\|\mathbf{G}^{\mathrm{H}}\mathbf{x}_n - \mathbf{s}_n\|^2\} \qquad (3)$$

with closed-form solution $\mathbf{G}_{\mathrm{MMSE}} = \mathbf{R}_x^{-1}\mathbf{H}$, in which $\mathbf{R}_x = \mathrm{E}\{\mathbf{x}\mathbf{x}^{\mathrm{H}}\}$ represents the sensor-output covariance matrix.

4.2 Optimal delay selection

Detector $\mathbf{G}_{\mathrm{MMSE}}$ extracts C time-shifted versions of each of the K inputs. However, the recovery of a single time delay per input suffices in practice. The time redundancy introduced by the wideband multipath channel in the MIMO model (2) enables the choice of the equalization delay providing the best MMSE performance for each input [7–9]. This choice is simplified thanks to the channel matrix estimate obtained in the blind identification stage. The MMSE detector of the ith input signal, $1 \leqslant i \leqslant K$, with delay $0 \leqslant d \leqslant (C-1)$, is given by:

$$\mathbf{G}_{i,d} = \mathbf{R}_x^{-1}\mathbf{h}_{i,d} \qquad (4)$$

in which $\mathbf{h}_{i,d}$ denotes the $(Kd+i)$th column vector of channel matrix $\mathbf{H}$. The resulting MMSE is easily obtained as:

$$\mathrm{MMSE}_{i,d} = \mathrm{E}\{|\hat{s}_i(n) - s_i(n-d)|^2\} = 1 - \mathbf{h}_{i,d}^{\mathrm{H}}\mathbf{R}_x^{-1}\mathbf{h}_{i,d}. \qquad (5)$$

Optimum MMSE equalization for the ith input is thus achieved at delay:

$$d_i = \arg\min_{d} \mathrm{MMSE}_{i,d}. \qquad (6)$$

In practice, even if the channel matrix is perfectly known, finite-sample errors in the sensor covariance matrix make the estimation of the optimum delay a difficult task. As reported elsewhere in the literature (e.g., [6, 8]), we have observed in extensive experiments that when the channel is fairly well conditioned, the optimum delay usually falls around the centre of the observation window, that is, $d_i \approx d_m \stackrel{\triangle}{=} \lceil (C-1)/2 \rceil$. For ill-conditioned channels, the optimality of the middle delay does not hold any more. Nonetheless, in our simulations the middle-delay selection seems to outperform more sophisticated optimum-delay estimation procedures based, e.g., on the principal components or polynomial-expansion approximations [13] of the sensor-output covariance matrix.

5. ICA-ASSISTED DETECTION

5.1 Rationale and assumptions

The practical MMSE receiver can notably divert from the theoretical performance given by eqn. (5). A typical example of these deviations is the flooring effect observed as the noise level decreases, due to the sampling noise caused by finite observation length. This effect will be illustrated by the results of Sec. 6. Even under perfect channel knowledge, imperfections in the covariance matrix estimate lead to residual interference in the equalized output, thus degrading detection performance.

In a bid to alleviate these detrimental effects, let us consider an additional hypothesis:

(A5) the input components are mutually statistically independent and consist of non-Gaussian i.i.d. data symbols.

Under this spatio-temporal independence assumption, eqn. (2) corresponds to a signal separation model of independent sources in instantaneous linear mixtures. This separation problem could be directly solved with the statistical tool of ICA based on higher-order statistics (HOS) [14]. The use of HOS requires the non-Gaussian assumption, which is verified by most digital modulations of practical significance. Unfortunately, the computational complexity of blindly separating $D = K(M+N)$ independent components can become excessive in scenarios with a large number of inputs or long delay spreads as a result of high data rates [11], particularly if the separation method is not properly initialized.

As suggested in the CDMA scenario of [10], the practical MMSE solution can be used as starting point in the ICA search. This sensible initialization, if close to the ICA solution, would generally reduce the number of iterations required for convergence. From another perspective, the ICA stage can be seen as a higher order refinement of the MMSE receiver, which is only (implicitly) based on SOS. Indeed, the MMSE-initialized ICA solution outperforms the practical MMSE detector [10, 15]. Despite the judicious initialization, the computational cost of extracting all independent signal components remains prohibitive in more realistic time-dispersive scenarios [15].

5.2 Single-delay signal extraction

To reduce complexity, the ICA algorithm may be tuned to search only for the K independent components associated with the input signals at their respective optimum MMSE delays. Let the corresponding columns of the estimated channel be stored in matrix $\mathbf{H}_K = [\mathbf{h}_{1,d_1}, \mathbf{h}_{2,d_2}, \ldots, \mathbf{h}_{K,d_K}]$. The MMSE equalizer of the components of $\mathbf{s}_n$ at the chosen delays accepts the equivalent expression

$$\mathbf{G}_K = \mathbf{W}\mathbf{H}_K \qquad (7)$$

which is to be applied on the whitened sensor output $\mathbf{z}_n = \mathbf{W}\mathbf{x}_n$, with $\mathbf{W} \in \mathbb{C}^{D\times P}$ such that $\mathbf{R}_z = \mathbf{I}_D$. Matrix equalizer (7) and the whitened data are used as the the initial point in the ICA algorithm. Final detection is then performed with the separating matrix $\mathbf{G}_K'$ provided by the ICA algorithm after convergence. We choose the fixed-point FastICA algorithm based on kurtosis optimization [14] for its robustness and rapid (cubic) convergence properties; not less importantly, the method is well adapted to the extraction of a particular group of source components. The specific algorithm used in the experiments reported later in this paper (Sec. 6) is detailed in [15].

5.3 Computational complexity

For a batch of T observed vectors $\mathbf{x}_n$, the computational complexity of the proposed ICA-based detection algorithm is of order $O(KDT)$ floating point operations (flops) per iteration. If the channel estimate is accurate enough, the above initialization may already be quite close to the ICA solution, thus reducing the number of iterations necessary for convergence. By contrast, extracting all independent components through the fully-blind application of ICA would require $O(D^2T)$ flops over a potentially higher number of iterations due to an improper initialization.

5.4 Remarks

The notion of ICA-aided detection was originally proposed in [10] for a particular DS-CDMA model without co-channel users. The authors of [10] were interested in extracting a fixed-delay component of a single user of interest in a short delay-spread channel with maximum time dispersion of half symbol period.

In contrast, the MIMO model of eqns. (1)–(2) characterizes a more general communication environment in which co-channel users transmit at the same time-frequency-code slot through a wideband channel with a possibly important delay spread. We aim at the simultaneous demodulation of all existing users, including all the spatially-multiplexed data substreams of each user in the case where multiple transmit antennas are employed. Time dispersion introduces undesired ISI, but can also be exploited to our own benefit: long delay spreads allow us to estimate the input signals at alternative equalization delays, which can lead to significant performance improvements, as demonstrated by the following experimental results.

6. EXPERIMENTAL RESULTS

A few simulations are useful in illustrating the performance improvements that can be achieved by the ICA-aided equalization scheme presented above, referred to as MMSE-ICA. A communication system composed of a number K of QPSK-modulated co-channel users with single transmit antennas is simulated in a frequency-selective block fading channel introducing ISI from a maximum of $M = 5$ consecutive baud periods. The channel filter taps are randomly drawn from a complex Gaussian distribution and hence model (up to the pulse-shaping and receive filters) a Rayleigh propagation environment. A spatio-temporal diversity level of $L = 20$ (e.g., 4 receive antennas oversampled by a factor of 5) and a smoothing factor of $N = 5$ are chosen. AWGN with covariance matrix $\mathbf{R}_v = \sigma^2\mathbf{I}_L$ is present at the sensor output, with

$$\text{SNR} = \frac{\text{trace}(\underline{\mathbf{H}}\,\underline{\mathbf{H}}^{\text{H}})}{\sigma^2 L}. \tag{8}$$

Equalization performance is measured by the signal mean square error (SMSE):

$$\text{SMSE} = \frac{1}{K}\sum_{i=1}^{K} \text{E}\{|\hat{s}_i(n) - s_i(n-\hat{d}_i)|^2\}. \tag{9}$$

Performance parameters are averaged over ν independent Monte Carlo (MC) iterations, with $\nu N_d \geqslant 10^4$, where N_d is the observation length in baud periods. The optimal MMSE denotes the optimum-delay MMSE equalizer obtained from the true sensor-output covariance matrix ($\hat{d}_i = d_i$). By MMSE, we refer to the practical subspace MMSE equalizer (7) in which the whitening matrix $\mathbf{W}$ is computed from the EVD of the covariance matrix sample estimate followed by projection on the signal subspace (whose dimension is assumed to be known) [4]. Both MMSE and MMSE-ICA extract the middle-delay components ($\hat{d}_i = d_m$).

Sample-size requirements. In the first experiments we simulate $K = 5$ users and the channel is assumed to be perfectly estimated. Fig. 1 plots the variation of equalization performance as a function of the sample size (measured in number of observed symbol periods) for various SNRs. For sufficient SNR, the MMSE-ICA requires substantially lower sample size than the MMSE for the same equalization performance and tends faster to the optimum theoretical solution. At moderate SNR, the MMSE-ICA needs just over two hundred symbol periods to start noticeably improving the conventional MMSE receiver.

Noise tolerance. The performance variation against SNR for various sample lengths is summarized in Fig. 2. The MMSE-ICA compensates from the sampling-noise flooring effect typical of the conventional MMSE receiver at high SNR, pushing performance closer to the theoretical bound. Equivalently, the MMSE-ICA tolerates higher noise power than the MMSE for the same equalization quality.

Capacity. Next, we illustrate the capacity gains that can be attained through ICA-assisted detection. Fig. 3 displays the equalization performance for a varying number of co-channel users, at several noise levels, with a fixed sample size of $N_d = 500$ baud periods. At 40 dB SNR, the proposed ICA-assisted detector allows a capacity increase of one order of magnitude relative to the conventional MMSE receiver.

Blindly identified channel. In a final simulation (again with $K = 5$ users), the channel is blindly identified with the MIMO extension of the SIMO method of [1] followed by ICA-based spatial-mixture separation (also performed with the FastICA algorithm), as explained in Sec. 3. Fig. 4 demonstrates the interesting fact that even when ICA takes part in channel identification, its use in detection also proves beneficial. Further results for blindly identified channels are reported in [15].

7. CONCLUSIONS

The assumptions of statistically-independent non-Gaussian i.i.d. inputs can be exploited to refine blind MIMO linear equalization through the use of ICA techniques based on HOS. The time dispersion introduced by the frequency-selective channel is beneficial in reducing the computational load while further improving performance by searching for the independent component with optimal MMSE reconstruction delay for each co-channel signal. In realistic SNR and sample-size conditions, the resulting ICA-assisted detector notably outperforms the conventional MMSE receiver, and is thus able to achieve remarkable capacity gains. As opposed to alternative detection approaches, the ICA-aided receiver is constellation independent. This feature makes ICA techniques very attractive in next-generation ad-hoc networks as well as in non-cooperative military scenarios. The phenomenal rate of increase in available computing power envisages the practical implementation of these techniques in the near future. Further work should focus on the theoretical analysis of the performance gains achieved by ICA-assisted detection.

ACKNOWLEDGEMENTS

V. Zarzoso carried out part of this work while on leave at Laboratoire I3S, Sophia Antipolis, France. He is grateful to Dr. Pierre Comon for his kind hospitality and interesting comments.

REFERENCES

[1] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization based on second-order statistics: a time domain approach," *IEEE Transactions on Information Theory*, vol. 40, no. 2, pp. 340–349, Mar. 1994.

[2] D. T. M. Slock, "Blind fractionally-spaced equalization, perfect-reconstruction filter banks and multichannel linear prediction," in *Proc. ICASSP-94, 19th International Conference on Acoustics, Speech and Signal Processing*, Adelaide, Australia, Apr. 19–22, 1994, vol. IV, pp. 585–588.

[3] E. Moulines, P. Duhamel, J.-F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Transactions on Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb. 1995.

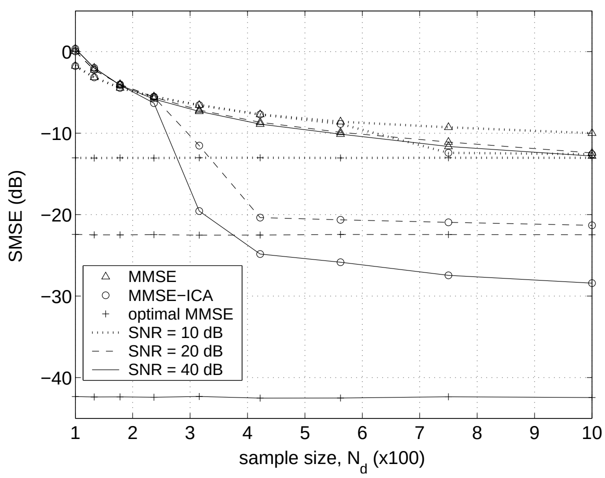

Figure 1: Equalization performance vs. sample size.

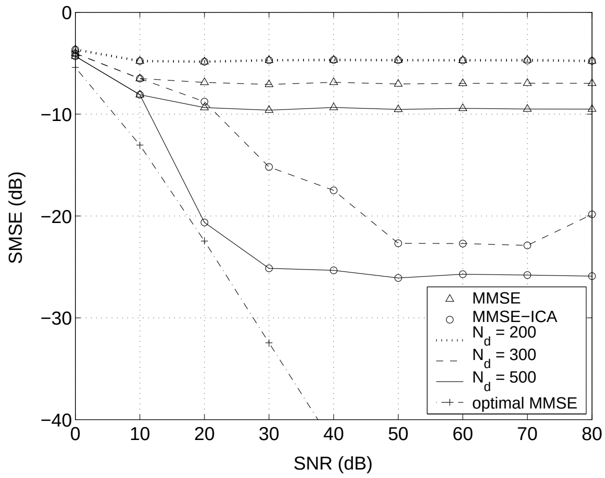

Figure 2: Equalization performance vs. SNR.

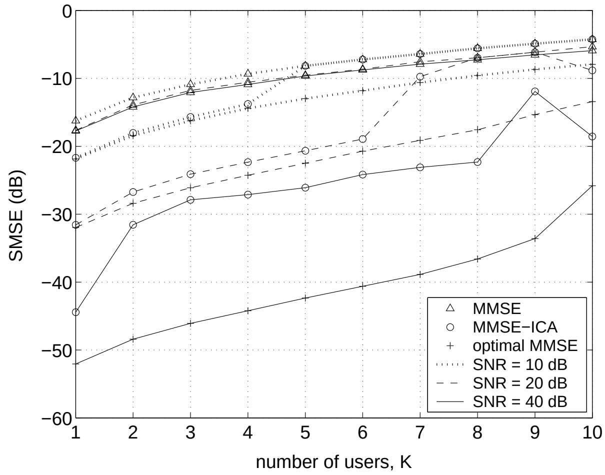

Figure 3: Equalization performance vs. number of users.

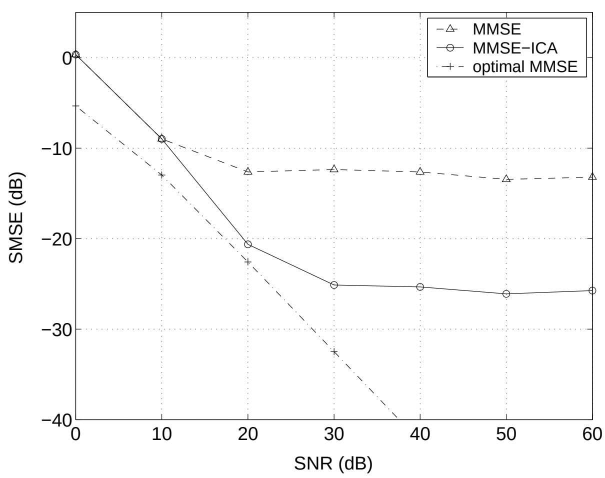

Figure 4: Blind equalization performance vs. SNR, with $N_d = 500$.

[4] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels," *IEEE Transactions on Communications*, vol. 46, no. 1, pp. 91–103, Jan. 1998.

[5] A.-J. van der Veen, S. Talwar, and A. Paulraj, "A subspace approach to blind space-time signal processing for wireless communication systems," *IEEE Transactions on Signal Processing*, vol. 45, no. 1, pp. 173–190, Jan. 1997.

[6] D. Gesbert, P. Duhamel, and S. Mayrargue, "Blind multichannel adaptive MMSE equalization with controlled delay," in *Proc. SSAP-96, IEEE Statistical Signal and Array Processing Workshop*, Corfu, Greece, June 24–26, 1996, pp. 172–175.

[7] G. B. Giannakis and S. D. Halford, "Blind fractionally spaced equalization of noisy FIR channels: direct and adaptive solutions," *IEEE Transactions on Signal Processing*, vol. 45, no. 9, pp. 2277–2291, Sept. 1997.

[8] J. Shen and Z. Ding, "Direct blind MMSE channel equalization based on second-order statistics," *IEEE Transactions on Signal Processing*, vol. 48, no. 4, pp. 1015–1022, Apr. 2000.

[9] H. Gazzah and K. Abed-Meraim, "Blind ZF equalization with controlled delay robust to order over estimation," *Signal Processing*, vol. 83, no. 7, pp. 1505–1518, July 2003.

[10] T. Ristaniemi and J. Joutsensalo, "Advanced ICA-based receivers for block fading DS-CDMA channels," *Signal Processing*, vol. 82, no. 3, pp. 417–431, Mar. 2002.

[11] V. Zarzoso and A. K. Nandi, "Exploiting non-Gaussianity in blind identification and equalization of MIMO FIR channels," *IEE Proceedings — Vision, Image and Signal Processing (Special Issue on non-Linear and non-Gaussian Signal Processing)*, vol. 151, no. 1, pp. 69–75, Feb. 2004.

[12] J. G. Proakis, *Digital Communications*, McGraw-Hill, New York, 4th edition, 2000.

[13] Z. D. Lei and T. J. Lim, "Simplified polynomial-expansion linear detectors for DS-CDMA systems," *Electronics Letters*, vol. 34, no. 16, pp. 1561–1563, Aug. 1998.

[14] A. Hyvärinen, J. Karhunen, and E. Oja, *Independent Component Analysis*, John Wiley & Sons, New York, 2001.

[15] V. Zarzoso and A. K. Nandi, "Blind MIMO equalization with optimum delay using independent component analysis," *International Journal of Adaptive Control and Signal Processing (Special Issue on Blind Signal Separation)*, vol. 18, no. 3, pp. 245–263, Apr. 2004.