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SHORT COMMUNICATION

Atrial activity estimation from atrial fibrillation ECGs by blind source extraction based on a conditional maximum likelihood approach

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Abstract This work presents a spatial filtering method for the estimation of atrial fibrillation activity in the cutaneous electrocardiogram. A linear extraction filter is obtained by maximising the extractor output power on the significant spectral support of the signal of interest. An iterative procedure based on a quasi-maximum likelihood estimator is proposed to jointly estimate the significant spectral support and the extraction filter. Compared with a previously proposed spatio-temporal blind source separation method, our approach yields an improved atrial activity signal estimate as quantified by a higher spectral concentration of the extractor output. The proposed methodology can readily be adapted to signal extraction problems in other application domains.

Keywords Blind source extraction · Atrial fibrillation · Second-order statistics · Conditional statistics · Electrocardiogram

1 Introduction

The present work deals with the estimation of the atrial electrical signal in the electrocardiogram (ECG) of patients suffering from atrial fibrillation (AF). The ECG consists of signal recordings obtained from cutaneous electrodes and reflects the spatio-temporal electrical activity of the heart as observed on the body surface. It is known that the contribution of AF measured in the ECG reveals underlying electro-physiological properties [2]. However, due to the simultaneous presence of atrial activity (AA), ventricular activity, respiration noise, etc. in the ECG, the electrophysiological parameters associated with AF cannot be estimated accurately from the ECG and clinical conclusions derived thereof might be erroneous.

A popular technique for the estimation of AF activity from the observed ECG is based on average beat subtraction [11, 12]. The underlying approach consists of the construction of a lead-specific template for the QRS(-T) complex by averaging over several complexes in a specific lead. This template is subsequently subtracted from each of the complexes in the observations by using an appropriate amplitude scaling factor and temporal alignment. The major drawback of these methods are their incapability to deal with observed waveforms that deviate from the template waveform, since the model assumes a stationary waveform observed in each complex with zero-mean noise. Also, the remainder contains all activity except the complexes and thus includes the AF waveforms, but also nuisance signals resulting from other physiological processes and noise (power line noise, electromagnetic interferences, etc.). In addition, the average beat subtraction approach is unable to efficiently exploit the spatial diversity available in the multi-lead ECG, that is, the fact that each lead captures the electro-physiological phenomena under study from a different spatial position.

In this work, we propose a direct approach to the estimation of the AF activity from the ECG based on spatial filtering and the narrowband character of the atrial signal. After transforming the observations into the frequency domain, we resort to the method of maximal variance in the

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conditional distribution tails (MaxViT), a generic source extraction algorithm recently proposed in [10]. The MaxViT method estimates the extraction filter by maximising a quasi-conditional likelihood criterion conditioned on a presence indicator associated with the source of interest. In the context of AA extraction, the presence indicator corresponds to the significant spectral support of the atrial signal, so that the process is equivalent to the maximization of the output-signal spectral concentration (SC) in the AF band. However, when the presence indicator is not explicitly available, as is the case in our problem, or is badly estimated, the estimation quality can deteriorate considerably. To overcome this difficulty, we offer here a solution to the estimation of the AF contribution in the ECG by jointly estimating the presence indicator (the spectral support of the AF) and the extraction filter.

2 Methods

2.1 Blind source extraction

In what follows, we will focus on source extraction methods to solve the AA estimation problem. Source extraction exploits explicitly the spatial diversity by searching for spatial filters (i.e. specific linear combinations of the lead outputs) that yield an estimate of the source of interest. Since the ECG electrode potentials are quasi linear with respect to the electrical cardiac activity for a spatially fixed source configuration [9], we may assume the following instantaneous, linear mixture model for the observed ECG, $\mathbf{y} \in \mathbb{R}^M$:

$$\mathbf{y} = \mathbf{A}\mathbf{s} \tag{1}$$

where $\mathbf{s} \in \mathbb{R}^N$ are the source signals and $\mathbf{A} \in \mathbb{R}^{M\times N}$ is a full column rank mixing matrix ($M \geq N$). The latent variables in the model are both the mixture matrix $\mathbf{A}$ and the sources $\mathbf{s}$.

A common strategy to estimate the source of interest, s_j, consists of obtaining a full separation of the observations into the sources $\mathbf{s}$, followed by a posterior selection. The full separation problem is based on a search for $\mathbf{H}^T \in \mathbb{R}^{N\times M}$ as an estimate for $\mathbf{A}^{-1}$ (or its pseudo-inverse if $M \neq N$). To narrow down the class of admissible solutions for $\mathbf{H}$, a source model needs to be imposed on $\mathbf{s}$. The most frequent source models in literature are the uncorrelated Gaussian distributed sources without temporal structure [8], mutually uncorrelated, spectrally coloured sources with non-proportional spectra [1, 14] or the mutually independently distributed sources [5, 7]. Since (linear) independence is not affected by scaling or permutation, the sources $\mathbf{s}$ can only be estimated up to the ambiguities $\mathbf{\Pi\Lambda s}$, where $\mathbf{\Pi} \in \mathbb{R}^{N\times N}$ and $\mathbf{\Lambda} \in \mathbb{R}^{N\times N}$ are, respectively, a permutation matrix and a non-degenerate diagonal scaling matrix. Independent Gaussian sources can only be estimated up to a more general rotation matrix. If $\mathbf{H}^T$ is a matrix of separation filters, we thus obtain $\mathbf{x} = \mathbf{H}^T\mathbf{A}\mathbf{s} = \mathbf{\Pi\Lambda s}$. Focussing on the estimation of s_j only, we thus need to select or identify *a posteriori* that x_i which is the best estimation of our source of interest s_j (up to scale).

The above mentioned approaches to source separation are optimal when all of the sources belong to one and the same model family, but suboptimal for the separation of a mixture of sources when those sources do not all belong to the same model family [13]. This can intuitively be explained from the fact that independence measures do not take into account temporal structures and cannot deal with Gaussian distributed signals, whereas the exploitation of non-proportional spectra do not take into account the spectrally spread energy of temporally impulsive waveforms with a rapidly vanishing autocorrelation function.

Unfortunately, the co-existence of temporally autocorrelated Gaussian sources and impulsive non-Gaussian sources is common in bio-electrical measurements. A typical example are the ECG signals recorded during AF episodes, where the impulsive ventricular activity (QRS-T complex) is superimposed to the AA contribution which has a significant autocorrelation function; see Fig. 1. It has already been shown in [4] that neither the method of second order blind identification (SOBI) [1], nor the method of independent component analysis (ICA) [5, 7] alone can estimate the atrial contribution. A possible solution has been proposed in [4], introducing the spatio-temporal blind source separation (ST-BSS) as a two stage, hybrid algorithm. First, source estimates are obtained using ICA. Next, the source subspace of the near Gaussian sources are estimated by putting an upper threshold on the normalised excess kurtosis values of the ICA components. This subspace is then decomposed into its source estimates by means of SOBI. Finally, the estimates resulting from the

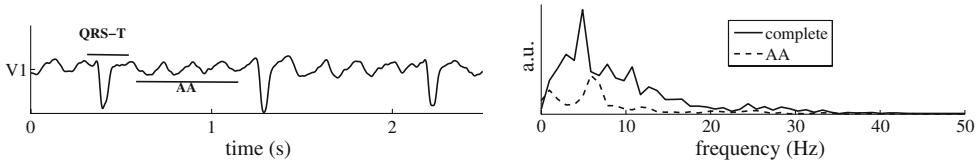

Fig. 1 [*Top*] A 2 s ECG fragment and [*Down*] its spectrum obtained by the discrete Fourier transform. The spectrum of the AA has been estimated by the presented method

latter stage, having a peak in the 3–9 Hz band, are selected as the activities related to AF. Unfortunately, the choice of the threshold is empirical and needs to be tuned to the data in hand. The authors propose a general threshold of 1.5 subject to inter- and intrapatient variability. In addition, the two-stage approach is not immune to propagating errors from the ICA subspace estimation stage to the SOBI component estimation stage.

On the other hand, most of the source separation algorithms are based on a pre-whitening stage or alternate between signal estimation and deflation of the observation space. While the pre-whitening based algorithms propagate the errors introduced by a whitening of the observations to the final processing stage [3], the estimation–deflation strategy does not generally guarantee the extraction of the source of interest before all other sources, so that the estimate x_j accumulates the errors over all the previous estimates x_i, $\forall i < j$ [6]. In this work, we propose to aim directly at the source of interest. Moreover, neither deflation, nor pre-whitening are needed, alleviating the limitations imposed by these processing steps.

2.2 Spectral concentration

We aim at estimating a spatial filter $\mathbf{h} \in \mathbb{R}^{\mathbb{M}}$, yielding $x = \mathbf{h}^T\mathbf{y}$ as an estimate for s_j. Let us define f_m as the modal frequency of the AF, f_s the sampling frequency, $\tilde{\mathbf{u}}(f)$ as the Fourier transform of a vector valued time series $\mathbf{u}(t)$ evaluated at the frequency f, $\mathbf{\Phi}_{\mathbf{u}}^{[a,\,b]} = \Re\left[\int_a^b \tilde{\mathbf{u}}(f)\tilde{\mathbf{u}}^H(f)\mathrm{d}f\right]$ and $\mathbf{\Phi}_{\mathbf{u}} = \mathbf{\Phi}_{\mathbf{u}}^{[0,\,f_\mathrm{s}/2]}$. The SC of x is given by

$$\mathrm{SC}(\tilde{x}) = \frac{\mathbf{\Phi}_x^{[0.82f_\mathrm{m},1.17f_\mathrm{m}]}}{\mathbf{\Phi}_x} \tag{2}$$

In [4], the SC from Eq. (2) has been proposed as a performance evaluation parameter for the estimation of AF. This is justified because the spectrum of AA is highly concentrated around the modal frequency, whereas the other contributions in the ECG typically have a larger bandwidth. Hence, SC will generally be maximal only if x contains no more than the AF contribution. According to this idea, we could use Eq. 2 as an objective function to estimate the extraction filter, since we have $\mathbf{\Phi}_x^{[0.82f_m,1.17f_\mathrm{m}]} = \mathbf{h}^T\mathbf{\Phi}_{\mathbf{y}}^{[0.82f_\mathrm{m},1.17f_\mathrm{m}]}\mathbf{h}$ and $\mathbf{\Phi}_x = \mathbf{h}^T\mathbf{\Phi}_{\tilde{\mathbf{y}}}\mathbf{h}$ from the linearity of the Fourier transform, and thus $\hat{\mathbf{H}} = \arg\max_{\mathbf{H}} \mathrm{SC}(\mathbf{H}^T\mathbf{y})$. However, maximising Eq. 2 is only possible if f_m is known. Unfortunately, f_m is generally not available as prior information and that explains the difficulty in using Eq. 2 in the estimation of s_j.

2.3 SC as a MaxViT contrast

Interestingly, the SC in Eq. 2 has the form of the MaxViT contrast function presented in [10]. Define by $\mathbb{E}\{\cdot\}$ the mathematical expectation and by $\mathbb{E}\{\cdot|e\}$ the mathematical expectation conditioned on the event e. In short, the theory of MaxViT states that

$$\Psi(x) = \frac{\mathbf{h}^T\Re\left[\mathbb{E}\left\{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H|\mathbb{I}_{\tilde{s}_j}\right\}\right]\mathbf{h}}{\mathbf{h}^T\Re[\mathbb{E}\{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H\}]\mathbf{h}} \tag{3}$$

with

$$\begin{cases} \mathbb{I}_{\tilde{s}_j} & \text{if } |\tilde{s}_j| \geq C \\ \bar{\mathbb{I}}_{\tilde{s}_j} & \text{otherwise} \end{cases}$$

is a contrast for the extraction of s_j for any arbitrary constant $C > 0$ under the following conditions:

C1: $\Re\left[\mathbb{E}\{\tilde{s}_j\tilde{s}_i^*\}\right] = 0, \forall i \neq j$
C2: $\Re\left[\mathbb{E}\{\tilde{s}_j\tilde{s}_i^*|\mathbb{I}_{\tilde{s}_j}\}\right] = 0, \forall i \neq j$
C3: $\frac{\Re\left[\mathbb{E}\{|\tilde{s}_j|^2|\mathbb{I}_{\tilde{s}_j}\}\right]}{\Re\left[\mathbb{E}\{|\tilde{s}_j|^2\}\right]} \geq \frac{\Re\left[\mathbb{E}\{|\tilde{s}_i|^2|\mathbb{I}_{\tilde{s}_j}\}\right]}{\Re\left[\mathbb{E}\{|\tilde{s}_i|^2\}\right]}, \forall i \neq j.$

In addition, the determination of $\hat{\mathbf{h}}$ through the MaxViT contrast can be carried out algebraically as the major generalised eigenvector of $\Re[\mathbb{E}\{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H\}]^{-1}\Re[\mathbb{E}\{\tilde{\mathbf{y}}\tilde{\mathbf{y}}^H|\mathbb{I}_{\tilde{s}_j}\}]$. Remark that the above conditions are also fulfilled, in particular, if the variable s_j is independently distributed with respect to s_i, $\forall i \neq j$ [10]. This is a reasonable assumption when we consider the AA with respect to the other contributions in the ECG during AF. Remark that the MaxViT criterion (Eq. 3) and the SC index (Eq. 2) become equivalent if the significant atrial frequency band, that is, the frequencies for which the atrial source $|\tilde{s}_j|$ verifies $\mathbb{I}_{\tilde{s}_j}$ corresponds to the interval $[0.82f_\mathrm{m},\ 1.17f_\mathrm{m}]$. In practice, however, the value of f_m is unknown, so that the MaxViT method needs to be modified to estimate both the modal frequency (or the closely related indicator function $\mathbb{I}_{\tilde{s}_j}$) and the extraction filter.

2.4 MaxViT contrast for atrial signal extraction

As explained above, Eq. 2 cannot be used as a contrast function, since we do not know f_m. However, we know that the AF activity has a rather compact support in the frequency domain, hence the use of SC as a performance parameter [4]. For a sampling frequency, f_s, the SC is defined on a relative bandwidth $\Delta f/(f_\mathrm{s}/2) = 0.25\ f_\mathrm{m}/(f_\mathrm{s}/2) = 0.5\ f_\mathrm{m}/f_\mathrm{s}$. Assume we have F frequency samples in $[0, f_\mathrm{s}/2]$, then SC is calculated on $S = 0.5\ f_\mathrm{m}/f_\mathrm{s} \times F = (1 - \alpha) \times F$ samples. Now, if we want to render S independent from f_m, we could choose α such that S is higher or equal than the number of samples used in the calculation of SC for $f_\mathrm{m} = 9$ Hz; e.g. for $f_\mathrm{s} = 1$ kHz, a good choice would be $\alpha = 0.995$. Interestingly, parameter α can be linked to a presence indicator $\mathbb{I}_{\tilde{s}_j}$. Consider

$$\begin{cases} \mathbb{I}_{\tilde{s}_j} & : \quad |\tilde{s}_j| \geq \gamma_\alpha(|\tilde{s}_j|) \\ \bar{\mathbb{I}}_{\tilde{s}_j} & : \quad |\tilde{s}_j| < \gamma_\alpha(|\tilde{s}_j|) \end{cases} \tag{4}$$

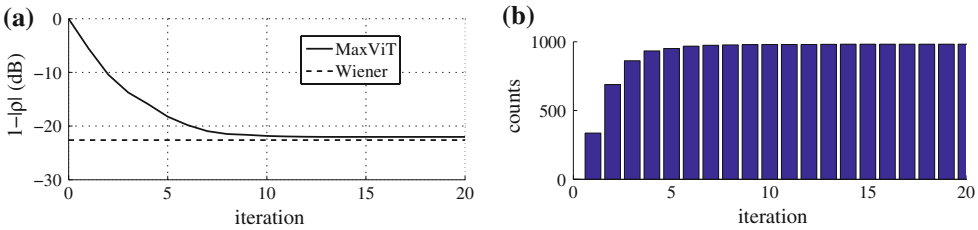

Fig. 2 The results as obtained on a synthetic dataset. **a** $1 - |\rho(x, s_{AF})|$ as a function of the iteration number and **b** the number of realisations for which $1 - |\rho(x, s_{AF})| < -20$ dB as a function of the iteration number

where $\gamma_\alpha(|\tilde{s}_j|)$ is the α-th ($0 \leq \alpha \leq 1$) percentile with respect to the distribution $p(|\tilde{s}_j|)$, defined as $\alpha = \int_0^{\gamma_\alpha(|\tilde{s}_j|)} p(|\tilde{s}_j|)\mathrm{d}|\tilde{s}_j|$. Hence, $\gamma_\alpha(|\tilde{s}_j|)$ can be seen as an adaptive threshold. Assuming $x \approx s_j$, then we may consider an estimation $\hat{\mathbb{I}}_{\tilde{s}_j}$ of $\mathbb{I}_{\tilde{s}_j}$ by replacing $\gamma_\alpha(|\tilde{x}|)$ for $\gamma_\alpha(|\tilde{s}_j|)$ in Eq. 4.

The above leads us to the following iterative procedure for the simultaneous estimation of the significant atrial frequency band and the extraction filter, somewhat inspired by the reference-based blind extraction method of [15]. The typical AF frequency band (3, 9) Hz is first divided into eight equal subbands in a logarithmic scale. Each of these subbands is used as an initial guess for the presence indicator $\hat{\mathbb{I}}_{\tilde{s}_j}^{[0]}$ of the AA. Using this guess, we compute the frequency-domain MaxViT estimate, which yields an extractor output $\tilde{x}^{[1]} = (\hat{\mathbf{H}}^{[1]})^T\tilde{\mathbf{y}}$. From $\tilde{x}^{[1]}$, we obtain $\hat{\mathbb{I}}_{\tilde{s}_j}^{[1]}$ according to the percentile-based procedure explained in the previous paragraph, and the MaxViT estimator is applied again to maximise the power spectral density of the extractor output associated to the new spectral support. By repeating this iteration until convergence, one atrial estimate per initial subband is obtained. Among them, we retain that with highest SC [Eq. 2].

3 Results

3.1 Synthetic data

As a synthetic dataset we use 12 second windows of 12-lead ECG ($f_s = 1$ kHz) recorded during normal sinus rhythm on which we superimpose simulated AF activity s_{AF}. The AF activity has been created along the lines of [12]. We choose f_m (f_0 in [12]) uniformly from (4, 8) Hz, whereas all other parameters in the model are chosen randomly within the same order of magnitude as the originally specified parameters in [12]. The synthetic s_{AF} is mixed into the observation by a random mixing vector $\mathbf{a}_{AF}$ with i.i.d. entries drawn from a Gaussian distribution. The estimation of s_{AF} is evaluated through the measure $1 - |\rho(x, s_{AF})|$, where ρ is Pearson's correlation coefficient. As a comparison, we also include the solution obtained by the optimal Wiener filter, given by $s_{AF}^{Wiener} = \mathbb{E}\{s_{AF}\mathbf{y}\}^T \mathbb{E}\{\mathbf{y}\mathbf{y}^T\}^{-1}\mathbf{y}$, which is the best possible linear estimator in the mean squared error sense, when the signal of interest is available as a reference. In Fig. 2a, the mean over 1,000 Monte Carlo realisations is shown for the respective frequency bands that result in the best estimate (evaluated through its SC) after 20 iterations. In Fig. 2b we display the number of realisations for which we reached the separation state, i.e. $1 - |\rho(x, s_{AF})| < -20$ dB, as a function of the iteration number.

3.2 Patient data

Electrocardiogram (ECG) data from 30 patients ($f_s = 1$ kHz, 12 leads) have kindly been made available to us by the Hemodynamics Department of Valencia University Hospital, Spain and ITACA-Bioingenieria, Polytechnic University of Valencia, Spain. We apply the same algorithm to this dataset and compare the obtained SC with the results by the ST-BSS method presented in [4]. In Fig. 3, we give the results as $\Delta SC = SC^{[MaxViT]} - SC^{[STBSS]}$ and $\Delta f_m = f_m^{[MaxViT]} - f_m^{[STBSS]}$. We have $\overline{\Delta SC} = 5.27\%$ and $\sigma_{\Delta SC} = 14.25\%$.

To show the convergence of the joint estimation of the atrial spectral presence indicator $\mathbb{I}_{\tilde{s}_j}$ and the filter $\mathbf{h}$, an illustrative example is given in Fig. 4.

4 Discussion

From Fig. 3, we observe that the MaxViT estimate generally offers a higher SC than the estimate obtained with the method of [4] and this for a similar modal frequency. The SC can be used as a performance measure for the estimation of the AF contribution in the ECG as it is correlated with the AA estimation quality in the synthetic recordings Castells et al. [4]. Hence, our technique proves

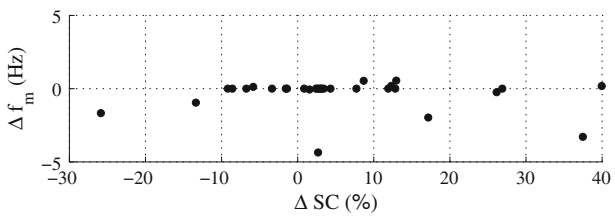

Fig. 3 Results on real data. The obtained SC and f_m from the estimate of MaxViT with respect to those obtained from the estimate of the hybrid method of [4]

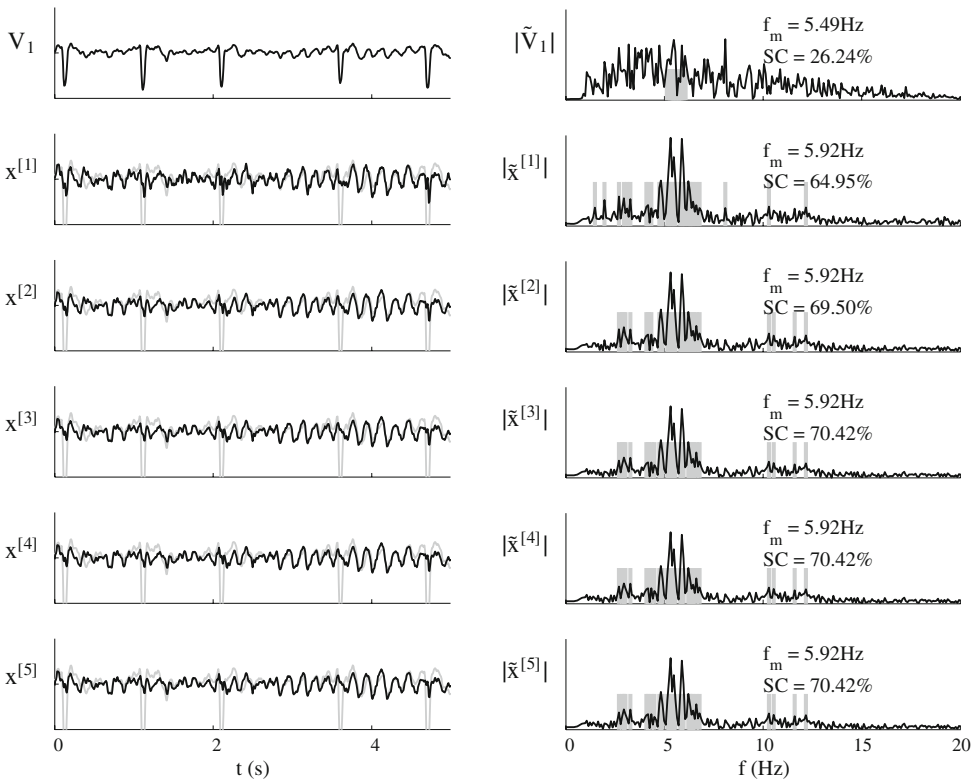

Fig. 4 The iterative version of MaxViT applied to real data. (*Left*) Time courses of the estimates scaled to V1. (*Right*) the corresponding amplitude spectrum and the estimated frequency set $\mathbb{I}_{\tilde{s}_j}$ in *gray* (see text)

superior in the majority of cases. Moreover, this increase in quality can be obtained with a decrease in computational complexity.

The optimisation strategy chosen in this paper is somewhat related to that of Xerri and Borloz [15], but retains all the benefits of the MaxViT technique [10]. More precisely, by contrast to [15], our approach can deal with any source probability densities, can target the specific source of interest, and does not require any correction term for certain source distributions.

5 Conclusion

The proposed method estimates the AF contribution in the ECG by maximising the power of the spatial filter output on the significant spectral support of the source of interest. The estimated signal obtained with the iterative version of the quasi-maximum likelihood based MaxViT method generally shows a higher SC than the estimate obtained from a related BSS-based two-stage hybrid method, and is thus associated with an improved atrial signal estimation quality. As a consequence, the estimated atrial contribution is expected to give rise to refined spatial, spectral and temporal characterisation of AF, which should be the subject of further research.

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